Solution (#868) Let

$$A = \begin{pmatrix} 1 & 3 & 2 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 2 & 4 & 1 \\ 0 & 2 & 1 & 3 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 7 \\ 2 \\ 1 \end{pmatrix}.$$

Note that

$$\begin{aligned} \det A &= [A]_{11}C_{11} + [A]_{12}C_{12} + [A]_{13}C_{13} + [A]_{14}C_{14} \\ &= 1 \times (-10) + 3 \times (-20) + 2 \times 10 + 1 \times 10 = -40. \end{aligned}$$

Cramer's Rule gives

$$\begin{aligned} x_1 &= \frac{-1}{40}\left[C_{11}b_1 + C_{21}b_2 + C_{31}b_3 + C_{41}b_4\right] \\ &= \frac{-1}{40}\left[(-10)(2) + (-19)(7) + (8)(2) + (7)(1)\right] \\ &= \frac{-130}{-40} = \frac{13}{4}. \end{aligned}$$

Similarly

$$\begin{aligned} x_2 &= \frac{-1}{40}\left[(-20)(2) + (6)(7) + (8)(2) + (2)(1)\right] = -\frac{1}{2} \\ x_3 &= \frac{-1}{40}\left[(10)(2) + (3)(7) + (-16)(2) + (1)(1)\right] = -\frac{1}{4} \\ x_4 &= \frac{-1}{40}\left[(10)(2) + (-5)(7) + (0)(2) + (-15)(1)\right] = \frac{3}{4}. \end{aligned}$$