Solution (#870) Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}.$$

(ii)

$$\mathrm{adj}A = \begin{pmatrix} -1 & 2 & -1 \\ 2 & -4 & 2 \\ -1 & 2 & -1 \end{pmatrix}.$$

Check that $(\mathrm{adj}A)A = 0_{33}$ and similarly that $A(\mathrm{adj}A) = 0_{33}$.
(iii) Suppose that a 3×3 matrix B satisfies $BA = AB = 0_{33}$ and has rows $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$. Explain why

$$\mathbf{r}_1 \cdot (1,2,3)^T = 0 \qquad \text{and} \qquad \mathbf{r}_1 \cdot (2,3,4)^T = 0,$$

and that the only such vectors are multiples of $(-1,2,-1)$. So B necessarily has the form

$$\begin{pmatrix} -a & 2a & -a \\ -b & 2b & -b \\ -c & 2c & -c \end{pmatrix}$$

for some values of a, b, c. However we also require that $AB = 0_{33}$ from which we can show that $B = t\mathrm{adj}A$ for some t as required.