Solution (#877) Let

$$A=\begin{pmatrix}0&1\\1&0\end{pmatrix}.$$

Suppose we had that

$$\begin{pmatrix}0&1\\1&0\end{pmatrix}=\begin{pmatrix}1&0\\l_{21}&1\end{pmatrix}\begin{pmatrix}u_{11}&u_{12}\\0&u_{22}\end{pmatrix}.$$

As

$$\begin{pmatrix}1&0\\l_{21}&1\end{pmatrix}^{-1}=\begin{pmatrix}1&0\\-l_{21}&1\end{pmatrix},$$

then we'd have

$$\begin{pmatrix}u_{11}&u_{12}\\0&u_{22}\end{pmatrix}=\begin{pmatrix}1&0\\-l_{21}&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}=\begin{pmatrix}0&1\\1&-l_{21}\end{pmatrix}.$$

But we then see that $u_{11}=0$ and that would be mean that U is singular and so $A=LU$ is singular, but this is clearly not the case and we have the desired contradiction.