

Solution (#879) Let A be a symmetric, invertible matrix with decomposition $A = LU$.

In a unique way, we can write $U = D\tilde{U}$ where D is a diagonal matrix and \tilde{U} is an upper triangular matrix with 1s on the diagonal. As A is symmetric then we have

$$A = A^T = (LD\tilde{U})^T = \tilde{U}^T DL^T.$$

Note that \tilde{U}^T is a lower triangular matrix with 1s on the diagonal, and DL^T is upper triangular. That is

$$A = \tilde{U}^T (DL^T)$$

is also a LU decomposition. By the uniqueness shown in #878 we have that

$$DL^T = U \implies LD = U^T \implies L^{-1}U^T$$

is diagonal as required.