Solution (#897) Suppose that $\mathbf{a}$ and $\mathbf{b}$ are linearly independent vectors in $\mathbb{R}^3$. Then $\mathbf{a} \wedge \mathbf{b} \neq \mathbf{0}$.

Say that

$$\alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{a} \wedge \mathbf{b} = \mathbf{0}.$$

If we dot this with $\mathbf{a} \wedge \mathbf{b}$ then we see

$$\gamma \left|\mathbf{a} \wedge \mathbf{b}\right|^2 = 0$$

and hence $\gamma = 0$. But then

$$\alpha\mathbf{a} + \beta\mathbf{b} = \mathbf{0},$$

and as $\mathbf{a}$ and $\mathbf{b}$ are independent then $\alpha = \beta = 0$. The result follows.