Solution (#902) Let $\mathbf{a}$ be a non-zero vector in $\mathbb{R}^3$. Note that any multiple of $\mathbf{a}$ clearly satisfies the given equation. So assume now that this is not the case, or equivalently that $\mathbf{r} \wedge \mathbf{a} \neq \mathbf{0}$.

By Proposition 3.184 we have

$$\mathbf{0} = (\mathbf{r} \wedge \mathbf{a}) \wedge \mathbf{r} = (\mathbf{r} \cdot \mathbf{r})\mathbf{a} - (\mathbf{r} \cdot \mathbf{a})\mathbf{r}.$$

If we take the vector product with $\mathbf{a}$ we see

$$(\mathbf{r} \cdot \mathbf{a})(\mathbf{r} \wedge \mathbf{a}) = \mathbf{0}.$$

As $\mathbf{r} \wedge \mathbf{a} \neq \mathbf{0}$ then we have $\mathbf{r} \cdot \mathbf{a} = 0$ but then

$$(\mathbf{r} \cdot \mathbf{r})\mathbf{a} = \mathbf{0}.$$

As $\mathbf{a} \neq \mathbf{0}$ then $\mathbf{r} \cdot \mathbf{r} = 0$ and so $\mathbf{r} = \mathbf{0}$, but this contradicts $\mathbf{r} \wedge \mathbf{a} \neq \mathbf{0}$.

In conclusion the only solutions of the equation

$$(\mathbf{r} \wedge \mathbf{a}) \wedge \mathbf{r} = \mathbf{0}$$

are of the form $\mathbf{r} = \lambda \mathbf{a}$ where λ is a real number.