Solution (#918) Say A is an invertible $n \times n$ matrix. So in particular its eigenvalues are non-zero. Note that

$$\begin{aligned} c_{A^{-1}}(\lambda) &= \det(\lambda I - A^{-1}) \\ &= \det\left(-\lambda\left(\frac{1}{\lambda}I - A\right)A^{-1}\right) \\ &= (-\lambda)^n \det\left(\frac{1}{\lambda}I - A\right)\det A^{-1} \\ &= (-\lambda)^n \det A^{-1} c_A\left(\frac{1}{\lambda}\right). \end{aligned}$$

So

$$\begin{aligned} \lambda \text{ is an eigenvalue of } A^{-1} &\iff c_{A^{-1}}(\lambda) = 0 \\ &\iff \det(\lambda I - A^{-1}) = 0 \\ &\iff (-\lambda)^n \det A^{-1} c_A\left(\frac{1}{\lambda}\right) = 0 \\ &\iff c_A\left(\frac{1}{\lambda}\right) = 0. \end{aligned}$$

Hence the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.