

Solution (#920) (i) As the eigenvalues $2, 1, -1$ are distinct then A_1 is diagonalizable. We can take

$$P = \begin{pmatrix} 1 & 3 & 1 \\ 0 & -1 & 3 \\ 0 & -1 & -3 \end{pmatrix}.$$

(ii) The eigenvalues are $x = -1, 2, 2$ and the 2-eigenvectors are scalar multiples of $(1, 1, 1)^T$. So A_2 is not diagonalizable.

(iii) The eigenvalues are $x = 4, 1, 1$. We find A_3 is diagonalizable and we can take

$$P = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & -1 \end{pmatrix}.$$