Solution (#932) Let A be a square matrix.

(a) let $\mathbf{v}$ be a λ-eigenvector of A and $c \neq 0$. Then $c\mathbf{v} \neq \mathbf{0}$ and

$$A(c\mathbf{v}) = cA(\mathbf{v}) = c\lambda\mathbf{v} = \lambda(c\mathbf{v})$$

and so $c\mathbf{v}$ is a λ-eigenvector.

(b) Say $\mathbf{v}$ and $\mathbf{w}$ are independent λ-eigenvectors and c, d are not both zero. Then $c\mathbf{v} + d\mathbf{w} \neq \mathbf{0}$ and

$$\begin{aligned} A(c\mathbf{v} + d\mathbf{w}) &= cA\mathbf{v} + dA\mathbf{w} \\ &= c\lambda\mathbf{v} + d\lambda\mathbf{w} \\ &= \lambda(c\mathbf{v} + d\mathbf{w}). \end{aligned}$$

Hence $c\mathbf{v} + d\mathbf{w}$ is a λ-eigenvector.