Solution (#944) In Example 3.206 we defined

$$x_{n+1}=x_n-y_n, \qquad y_{n+1}=x_n+y_n, \qquad x_0=1,\, y_0=0.$$

If we set $z_n=x_n+iy_n$ then we have

$$\begin{aligned} z_{n+1} &= x_{n+1}+iy_{n+1} \\ &= (x_n-y_n)+i(x_n+y_n) \\ &= (1+i)(x_n+iy_n) \\ &= (1+i)z_n. \end{aligned}$$

As $z_0=1+0i=1$ then

$$z_n=(1+i)^n\,.$$

By De Moivre's theorem we have

$$z_n=\left(\sqrt{2}\text{cis}(\pi/4)\right)^n=2^{n/2}\text{cis}(n\pi/4)$$

and hence

$$x_n=2^{n/2}\cos(n\pi/4), \qquad y_n=2^{n/2}\sin(n\pi/4).$$