**Solution** (#963) Let $a, b$ be real numbers and define

$$(x_1, x_2) \cdot (y_1, y_2) = x_1 y_1 + a x_2 y_1 + b x_1 y_2 + y_1 y_2.$$

Note that linearity (IP1) holds true for all values of $a$ and $b$. For symmetry (IP2) we need

$$(x_1, x_2) \cdot (y_1, y_2) = (y_1, y_2) \cdot (x_1, x_2)$$

for all values of $x_1, x_2, y_1, y_2$. Note that we must have $a = b$ from

$$b = (1, 0) \cdot (0, 1) = (0, 1) \cdot (1, 0) = a,$$

and conversely that if $a = b$ then IP2 follows.

Finally for IP3 to holds we need

$$(x_1, x_2) \cdot (x_1, x_2) = x_1^2 + 2a x_1 x_2 + x_2^2 > 0$$

for all $x_1$ and $x_2$, not both zero. Setting $x_1 = x_2 = 1$ means

$$2 + 2a > 0 \quad \Longrightarrow \quad a > -1$$

and setting $x_1 = -x_2 = 1$ means

$$2 - 2a > 0 \quad \Longrightarrow \quad a < 1.$$

So necessarily we must have $-1 < a < 1$. If these inequalities hold then

$$\begin{aligned}(x_1, x_2) \cdot (x_1, x_2) &= x_1^2 + 2a x_1 x_2 + x_2^2 \\ &= (x_1 - x_2)^2 + 2(1 + a) x_1 x_2 \\ &= (x_1 + x_2)^2 + 2(a - 1) x_1 x_2.\end{aligned}$$

From one of these last two expressions we can see that $(x_1, x_2) \cdot (x_1, x_2) \geqslant 0$ depending on the sign of $x_1 x_2$ and further $(x_1, x_2) \cdot (x_1, x_2) = 0$ means $x_1 x_2 = 0$ and $x_1 = \pm x_2$ so that $x_1 = x_2 = 0$.

In conclusion $\cdot$ is an inner product if and only if $-1 < a < 1$.