

Solution (#964) Recall that

$$D(\mathbf{p}, \mathbf{q}) = \sum_{i=1}^n |p_i - q_i|; \quad d(\mathbf{p}, \mathbf{q}) = \sqrt{\sum_{i=1}^n (p_i - q_i)^2}; \quad \delta(\mathbf{p}, \mathbf{q}) = \max_{1 \leq i \leq n} |p_i - q_i|.$$

Note that

$$d(\mathbf{p}, \mathbf{q}) = \sqrt{\sum_{i=1}^n (p_i - q_i)^2} \geq \sqrt{\max_{1 \leq i \leq n} |p_i - q_i|^2} = \max_{1 \leq i \leq n} |p_i - q_i| = \delta(\mathbf{p}, \mathbf{q})$$

and that

$$D(\mathbf{p}, \mathbf{q})^2 = \left(\sum_{i=1}^n |p_i - q_i| \right)^2 \geq \sum_{i=1}^n |p_i - q_i|^2 = d(\mathbf{p}, \mathbf{q})^2.$$

Hence

$$\delta(\mathbf{p}, \mathbf{q}) \leq d(\mathbf{p}, \mathbf{q}) \leq D(\mathbf{p}, \mathbf{q}) \quad \text{for any } \mathbf{p}, \mathbf{q} \text{ in } \mathbb{R}^n.$$

The desired loci then look like

