Solution (#985) Let $V = \mathbb{R}^2$ and

$$X = \langle \mathbf{e}_1 \rangle, \qquad Y = \langle \mathbf{e}_2 \rangle, \qquad Z = \langle \mathbf{e}_1 + \mathbf{e}_2 \rangle.$$

Then $V = X + Y + Z$ as any vector $\mathbf{v}$ in $\mathbb{R}^2$ can be written as

$$\mathbf{v} = \underbrace{v_1 \mathbf{e}_1}_{\text{in } X} + \underbrace{v_2 \mathbf{e}_2}_{\text{in } Y} + \underbrace{\mathbf{0}}_{\text{in } Z}.$$

These three lines X, Y, Z each meet only in the origin. However if it were the case that

$$V = X \oplus Y \oplus Z$$

then it would follow that

$$\begin{aligned} 2 &= \dim \mathbb{R}^2 \\ &= \dim X + \dim Y + \dim Z \\ &= 1 + 1 + 1 \\ &= 3, \end{aligned}$$

a contradiction.