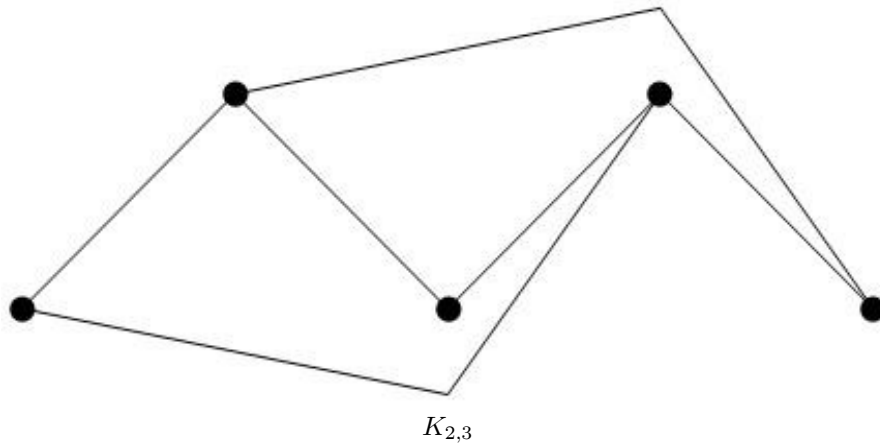


Solution (#1017) (i)



(ii) Suppose that $K_{3,3}$ is planar. Note that $K_{3,3}$ has 6 vertices and 9 edges. Further a face must have at least 4 edges as each edge connects the two sets of vertices, and only one edge connects any such pair of vertices from the two groupings. Thus we have

$$18 = 2E \geq 4F \implies F \leq 4.$$

But we then have

$$V - E + F \leq 6 - 9 + 4 \leq 1$$

which is a contradiction.

(iii) A torus can be formed by gluing opposite sides of a square, as directed by the single arrows and double arrows on the square below. So we can draw $K_{3,3}$ on a torus as below.

