Solution (#1026) Say that the graph G is bipartite with the vertex set V partitioned into V_1 and V_2 with each edge being between these two sets. Any walk

$$v_0 v_1 v_2 \dots v_k$$

alternates between V_1 and V_2. If that walk begins and ends in the same vertex, then $v_k = v_0$ lies in V_1 or $v_k = v_0$ lies in V_2. Either way k is even

Conversely, suppose that G has no walks of odd length beginning and ending in the same vertex. Fix a vertex v_0 and let V_1 be the set of vertices v such that there is an even length walk between v_0 and v and let V_2 be the set of vertices v such that there is an even length walk between v_0 and v. As we have assumed graphs to be connected then every vertex is in V_1 or V_2. Further no vertex v is in V_1 and V_2; were this the case there would be an even length walk from v to v_0 and an even length walk from v_0 to v. Together they would make an odd length walk that begins and ends in the same vertex – a contradiction.