

Decomposition numbers for symmetric groups

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Where to find decomposition matrices?

Try www.math.rwth-aachen.de/~MOC

In hard copy, [6] has several (at the end) for small primes.

For $p = 2$, a paper by Juergen Mueller(Aachen) [7], for an electronic version try his homepage.

General background can be found in [6].

Column removal: Due to G.D. James, sometime after '76. Just now I cannot find the precise reference. However, a generalization of the various removals has recently been done by Fayers and Lyle [3]. A reference to the original should be in there.

Two-part partitions: The original paper is [5], it is just for $p = 2$. The odd prime paper is slightly later (again, I cannot find the reference just now). But there is not much in it, he just says 'it's the same as $p = 2$.

His proof is quite long. There is now a short proof, due to S. Donkin, using new technology. This can be found in [1], the last chapter.

More on fractal structure of decomposition matrices for 2-part partitions can be found in [4].

Blocks. The result by Scopes (including details with the abacus) is in [8].

The decomposition map: The details are in the paper [2].

to be added reference for the number of simples in a p -block of weight w . Maybe in [8] there is a reference?

References

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