Axioms for Rings

A *ring* is a set R with distinguished element 0 and with two binary operations $+$ and $\times$ satisfying the following conditions. Conventionally

for the function $+ : R \times R \to R$ we write $(a, b) \mapsto a+b$;
for $\times : R \times R \to R$ we write $(a, b) \mapsto ab$.

(1) $a + (b + c) = (a + b) + c$ [+ is associative]

(2) $a + b = b + a$ [+ is commutative]

(3) $a + 0 = a$

(4) $(\forall a \in R)(\exists b \in R)(a + b = 0)$

(5) $a(bc) = (ab)c$ [× is associative]

(9) $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$
[× distributes over +]

If

(6) $ab = ba$ [× is commutative]

then the ring is said to be *commutative*.

If there exists $1 \in R$ such that

(7) $\forall a \in R : a1 = 1a = a$

Then R is a *ring with unity* or a *ring with* 1.