Problem sheet (To be done in week 2)
Linear Algebra I, Dr A Henke, MT 2007

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This problem sheet is for test purposes only. You are not obliged to solve it. However it is recommended that you make yourself familiar with the OKUSON software. The solutions to the following problems should be submitted via the web-interface.

1 In $\mathbb{M}_{4\times4}(\mathbb{R})$ consider the matrices $A=(a_{ij})$, $B=(b_{ij})$ and $C=(c_{ij})$ defined by:

$$a_{ij} := (-1)^{i+j-1}, \quad b_{ij} := \tfrac{i+1}{j}, \quad c_{ij} := \begin{cases} j-i & \text{if } i<j \\ i & \text{otherwise} \end{cases}$$

Enter c_{32}.	_____
Enter b_{34}.	_____
Enter c_{14}.	_____

2 Consider the following matrices with real coefficients:
$A := \begin{pmatrix} 14 & -13 \\ 1 & -19 \\ 5 & 6 \end{pmatrix}$, $B := \begin{pmatrix} 14 & -13 & 2 \\ 1 & -19 & 5 \end{pmatrix}$, $C := \begin{pmatrix} 4 & 3 & 2 \\ -1 & -2 & -3 \\ 9 & 12 & -24 \end{pmatrix}$, $D := \begin{pmatrix} 1 & -6 \\ -1 & 7 \end{pmatrix}$.
For each of the following expressions, decide whether the expression makes sense and defines a matrix $X=(x_{ij})$. If not, cross the box labelled N (for *nonsense*), otherwise cross the box labelled by the value of entry x_{11}.

$X = DBA - D^3$	◯ N / ◯ 79 / ◯ 18
$X = CAC$	◯ N / ◯ -97 / ◯ 188
$X = 4AB - 183C$	◯ N / ◯ 0 / ◯ 1

3 Which of the following matrices is diagonal, which is invertible?
$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $E = \begin{pmatrix} 8 & 4 \\ 2 & 1 \end{pmatrix}$.

Is D invertible?	◯ Yes / ◯ No
Is B invertible?	◯ Yes / ◯ No
Is E diagonal?	◯ Yes / ◯ No

4 Define the matrices A and I_2 by $A := \begin{pmatrix} 1 & -6 \\ -1 & 7 \end{pmatrix}$, $I_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
Define $f(x) = (x-I_2, x+I_2)$ and $g(x,y) = 3x-2y$ for $x,y \in \mathbb{M}_{2\times2}(\mathbb{R})$.

Calculate $X := g(A, I_2)$. Enter x_{21}.	_____
Calculate $X := (g \circ f)(A)$. Enter x_{11}.	_____
Calculate $X := (g \circ f)(I_2)$. Enter x_{11}.	_____

5 Define the matrices A and I_2 by $A := \begin{pmatrix} 1 & -6 \\ -1 & 7 \end{pmatrix}$, $I_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
Define $f : \mathbb{M}_{2\times2}(\mathbb{R}) \to \mathbb{M}_{2\times2}(\mathbb{R}) \times \mathbb{M}_{2\times2}(\mathbb{R})$ by $f(x) = (x-I_2, x+I_2)$.
Define $g : \mathbb{M}_{2\times2}(\mathbb{R}) \times \mathbb{M}_{2\times2}(\mathbb{R}) \to \mathbb{M}_{2\times2}(\mathbb{R})$ by $g(x,y) = 3x-2y$.
Answer the following questions.

	Is f surjective?	◯ Yes / ◯ No
	Is g surjective?	◯ Yes / ◯ No
	Is $g \circ f$ surjective?	◯ Yes / ◯ No
6	Define the following 2×2-matrices with real coefficients: $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}. D = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, E = \begin{pmatrix} 8 & 4 \\ 2 & 1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$ In each case, determine the number of maps with the requested properties.	
	The number of bijective maps from $\{A,B,C\}$ to $\{A,B,C\}$.	_____
	The number of surjective maps from $\{-C,-B,-A\}$ to $\{A,\{B,C\},C\}$.	_____
	The number of injective maps from $\{A,B,C\}$ to $\{D,E\}$.	_____
This demo problem sheet is not requesting you to do any written homework.		