Problem sheet (To be done in week 4)

Linear Algebra I, Dr A Henke, MT 2007

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Deadline for online homework: Sat Oct 27 18:00:00 2007
This file has been created: Wed Oct 10 16:40:57 2007

The points of this online-homework will be recorded. For every correct answer, you get 1 point; for every wrong answer, you get -1 points. For every question you decide not to answer (indicated by the option -) you get 0 points.		
1	In each of the following cases, decide whether V is a vector space over $\mathbb{R}$.	
	V is the set of all scalar matrices of size $n \times n$ over $\mathbb{R}$.	◯ Yes / ◯ No
	V is the set of all orthogonal $n \times n$-matrices over $\mathbb{R}$.	◯ Yes / ◯ No
	V is the set of all skew-symmetric $n \times n$ matrices over $\mathbb{R}$.	◯ Yes / ◯ No
	$V = \mathbb{R}^2$ with the usual scalar multiplication and the new addition $\oplus : V \times V \to V$ given by $$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + y_2 + 1 \\ x_2 + y_1 - 1 \end{pmatrix}.$$	◯ Yes / ◯ No
2	Which of the following sets U are subspaces of the given $\mathbb{R}$-vector spaces?	
	$U := \{A \in \mathbb{M}_{n\times n}(\mathbb{R}) \mid \mathrm{tr}(A) = 1\} \subseteq \mathbb{M}_{n\times n}(\mathbb{R})$	◯ Yes / ◯ No
	$U := \{A \in \mathbb{M}_{n\times n}(\mathbb{R}) \mid A = 2A\} \subseteq \mathbb{M}_{n\times n}(\mathbb{R})$	◯ Yes / ◯ No
	$U := \left\{(a_{ij}) \in \mathbb{M}_{4\times 3}(\mathbb{R}) \mid a_{ij} = 0 \text{ for } i+j \text{ even}\right\} \subseteq \mathbb{M}_{4\times 3}(\mathbb{R})$	◯ Yes / ◯ No
	$U := \left\{(a_{ij}) \in \mathbb{M}_{3\times 3}(\mathbb{R}) \mid a_{11}^2 + a_{22}^2 = 0\right\} \subseteq \mathbb{M}_{3\times 3}(\mathbb{R})$	◯ Yes / ◯ No
3	Let $V = \mathbb{R}[x]$, the vector space of all real polynomials in one variable x and U defined as below. Is U a subspace of V?	
	U consists of all polynomials $p(x) \in \mathbb{R}[x]$ with $p(1) + p(5) = p(6)$.	◯ Yes / ◯ No
	U consists of all polynomials $p(x) \in \mathbb{R}[x]$ with $2 \cdot p(1) = p(5)$.	◯ Yes / ◯ No
	U consists of all polynomials with degree $\geq k$ for fixed k, together with the zero polynomial;	◯ Yes / ◯ No
	U consists of all polynomials $p(x) \in \mathbb{R}[x]$ with $p(1) + p(5) = 0$.	◯ Yes / ◯ No
4	Given are subsets U, W of $\mathbb{M}_{1\times 3}(\mathbb{R})$. Decide whether the following statements are correct.	
	If $U = \{(0,x,0) \mid x \in \mathbb{R}\}$ and $W = \{(0,x,x) \mid x \in \mathbb{R}\}$, then $U + W = \{(0,y,x) \mid x,y \in \mathbb{R}\}$.	◯ Yes / ◯ No
	If $U = \{(0,x,0) \mid x \in \mathbb{R}\}$ and $W = \{(0,x,x) \mid x \in \mathbb{R}\}$, then $U \cup W = \{(0,2x,x) \mid x \in \mathbb{R}\}$.	◯ Yes / ◯ No
	If $U = \{(0,x,y) \mid x,y \in \mathbb{R}\}$ and $W = \{(0,x,x) \mid x \in \mathbb{R}\}$, then $U \cap W = W$.	◯ Yes / ◯ No
	If $U = \{(0,x,y) \mid x,y \in \mathbb{R}\}$ and $W = \{(0,x,x) \mid x \in \mathbb{R}\}$, then $U \cup W \supseteq W$.	◯ Yes / ◯ No
Please submit your written solutions to the following problems to your college tutor. For this written part of your homework, the deadline set by your college tutor applies.		

5	(a) Let V be the set of all functions $f : X \to \mathbb{R}$ (for some fixed non-empty set X), and if $f,g \in V$, $\alpha \in \mathbb{R}$, then the functions $f+g$, αf are defined by setting $$(f+g)(x) = f(x)+g(x), \quad (\alpha f)(x) = \alpha f(x).$$ Prove that V is a vector space over $\mathbb{R}$. (b) Let $\alpha \in \mathbb{R}$. Prove that $U_\alpha = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid x_1+x_2+x_3 = \alpha\}$ is a subspace of $\mathbb{R}^3$ if and only if $\alpha = 0$.
6	Let V be a vector space over $\mathbb{R}$. Use the vector space axioms to show that for all $v \in V$ and all $\lambda \in \mathbb{R}$ the following holds: $$\text{(a) } \lambda \cdot 0_V = 0_V \qquad\qquad \text{(b) } (-1)v = -v.$$
7	Let U,W be subspaces of a vector space V. Prove that (a) $U \cap W$ is a subspace of V; (b) $U+W = \{u+w \mid u \in U, w \in W\}$ is a subspace of V; (c) $U \cup W$ is a subspace of V if and only if $U \subset W$ or $W \subset U$.
8	(Optional.) (a) Let S be the subset $\{(x,0) \mid x \in \mathbb{R} \text{ and } x > 0\}$ of $\mathbb{R}^2$. Is S a subspace of the vector space $\mathbb{R}^2$ with respect to the usual scalar multiplication and the usual addition of $\mathbb{R}^2$? (b) Let S be the subset $\{(x,0) \mid x \in \mathbb{R} \text{ and } x > 0\}$ of $\mathbb{R}^2$. Define the scalar multiplication $*$ and addition $\oplus$ on S by: $$\alpha * (u,0) = (u^\alpha,0), \qquad (u,0) \oplus (v,0) = (uv,0)$$ for all $\alpha,u,v \in \mathbb{R}$ with $u,v > 0$. Show that S is a vector space with respect to $*$ and $\oplus$. Is $(S,*,\oplus)$ a subspace of $\mathbb{R}^2$?