Problem sheet (To be done in week 5)

Linear Algebra I, Dr A Henke, MT 2007

Student Number: 654321
Deadline for online homework: Sat Nov 3 18:00:00 2007
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The points of this online-homework will be recorded. For every correct answer, you get 1 point; for every wrong answer, you get -1 points. For every question you decide not to answer (indicated by the option -) you get 0 points. Submit your answers via the web-interface.		
1	Given is a vector space and a subset. Are the given subsets linearly independent?	
	$\{(1,2,3),(2,3,1),(3,1,2),(0,0,0)\} \subseteq \mathbb{M}_{1\times 3}(\mathbb{R})$	◯ Yes / ◯ No
	$\{(1,2,3),(3,1,2)\} \subseteq \mathbb{M}_{1\times 3}(\mathbb{R})$	◯ Yes / ◯ No
	$\{\pi\} \subseteq \mathbb{R}$	◯ Yes / ◯ No
2	Let $\mathbb{R}^{\mathbb{R}}$ be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$. Are the given subsets of $\mathbb{R}^{\mathbb{R}}$ linearly independent?	
	$\{-5+x+3x^2, 13+x, 1+x+2x^2\} \subseteq \mathbb{R}^{\mathbb{R}}$	◯ Yes / ◯ No
	$\{f,g,h\} \subseteq \mathbb{R}^{\mathbb{R}}$ with $f(x)=5x^2+x+1$, $g(x)=2x+3$ and $h(x)=x^2-1$	◯ Yes / ◯ No
	$\{\cos^2(x), 1\} \subseteq \mathbb{R}^{\mathbb{R}}$	◯ Yes / ◯ No
3	Given is a vector space V with basis $\{b_1,b_2,b_3\}$. Are the following sets bases of V?	
	$\{b_1-b_3, 2b_1+b_2, b_1-b_2-3b_3\}$	◯ Yes / ◯ No
	$\{b_1, b_1+b_2, b_1+b_2+b_3\}$	◯ Yes / ◯ No
	$\{b_1, b_2, b_1+b_2, b_1-b_3\}$	◯ Yes / ◯ No
4	Let A and B be matrices with real entries such that $A\cdot B$ is defined. Consider the columns and rows of matrices as vectors. Which of the following statements is correct?	
	Each column of $A\cdot B$ lies in the vector space spanned by the columns of A.	◯ Yes / ◯ No
	The rows of $A\cdot B$ are linear combinations of the rows of A.	◯ Yes / ◯ No
	The columns of $A\cdot B$ are linear combinations of the columns of B.	◯ Yes / ◯ No
5	Let V be a (finitely generated) vector space and $X \subseteq Y \subseteq V$. Are the following statements correct?	
	If Y is linearly independent, then also X is linearly independent.	◯ Yes / ◯ No
	If Y is a generating system of V, then also X is a generating system of V.	◯ Yes / ◯ No
	Assume there exists a subset $Y' \subseteq Y$ with $X \subseteq Y'$ such that Y' forms a basis of Span(Y), then X is a basis of Span(X).	◯ Yes / ◯ No
Please submit your written solutions to the following problems to your college tutor. For this written part of your homework, the deadline set by your college tutor applies.		
6	Determine all $\alpha \in \mathbb{R}$ for which the set $\{(1,\alpha,\alpha)$, $(\alpha,1,\alpha)$ and $(\alpha,\alpha,1)\} \subset \mathbb{R}^3$ is linearly independent.	

7	Let V be an $\mathbb{R}$-vector space, $n \in \mathbb{N}$ and $v_1, \ldots, v_n \in V$. Define vectors w_i for $1 \leq i \leq n$ by $$w_i = \sum_{j=1}^{i} v_j.$$ (a) Show that $\mathrm{Span}\{v_1, \ldots, v_n\} = \mathrm{Span}\{w_1, \ldots, w_n\}$. (b) Show that $\{w_1, \ldots, w_n\}$ is linearly independent if and only if $\{v_1, \ldots, v_n\}$ is linearly independent.
8	Show that the vectors $1, 1+x, 1+x+x^2, \ldots, 1+x+\ldots+x^n$ form a basis of $\mathbb{R}_n[x]$, the polynomials of degree at most n in one variable x.
9	(Optional.) Let V, W be vector spaces over $\mathbb{R}$. Consider the cartesian product $V \times W$ with componentwise addition and scalar multiplication: $$(v_1, w_1) + (v_2, w_2) = (v_1 + v_2, w_1 + w_2), \qquad \lambda \cdot (v_1, w_1) = (\lambda v_1, \lambda w_1)$$ for all $v_1, v_2 \in V, w_1, w_2 \in W$ and $\lambda \in \mathbb{R}$ (which defines on $V \times W$ a vector space structure). Let S be a basis of V and T be a basis of W. Give a basis for the vector space $V \times W$ and justify your answer.