Two-Dimensional Transonic Aerodynamic Design Method

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This paper demonstrates the capabilities of a new transonic, two-dimensional design method, based on the simultaneous solution of multiple streamtubes, coupled through the position of, and pressure at, the streamline interfaces. This allows the specification of either the airfoil shape (direct, analysis mode) or the airfoil surface pressure distribution (inverse, design mode). The nonlinear system of equations is formulated in a conservative manner, which guarantees the correct treatment of shocks, and is solved by a rapid Newton solution method. Viscous effects can also be included through a coupled integral boundary-layer analysis. The first set of results shows the effect of different far-field treatments, demonstrating the improvement in accuracy obtained by including the second-order doublet terms in addition to the usual first-order vortex term. The results are also compared to those obtained by specifying straight far-field streamlines (corresponding to solid-wall wind-tunnel experiments) or constant far-field pressure (corresponding to freejet experiments) to show the sensitivity to the far-field distance. In the second set of results, the design method is used to design a transonic airfoil with $C_l = 1.000$ at $M_\infty = 0.70$. It is shown that the off-design performance is improved by specifying a surface pressure distribution with a very weak shock.

Nomenclature

C_d = drag coefficient
C_l = lift coefficient
D_x, D_y = far-field doublet strengths
h_0 = stagnation enthalpy
m = mass flux
M_∞ = freestream Mach number
p = pressure (in streamtubes)
q = speed
$\mathbf{q}$ = velocity
Re = Reynolds number
$\mathbf{s} \equiv \mathbf{q}/q$ = unit vector in flow direction
x, y = coordinates
γ = ratio of specific heats
Γ = far-field vortex strength
Π = pressure (on streamlines)
ρ = density
Σ = far-field source strength
Φ = far-field potential

I. Introduction

MOST computational methods for predicting transonic flow over airfoils are analysis methods, which predict the surface pressure distribution on an airfoil with a specified geometry. Relatively few methods address the aerodynamic designer's task of designing more efficient airfoils. Those that do, fall into two categories.

The first category is optimization methods, which couple a conventional analysis method with an optimization algorithm to modify iteratively the geometry in order to minimize some "cost" function, such as the drag or the difference between the actual and desired surface pressure distributions. The methods of Vanderplaats[1] and Hicks[2] are examples of this category. The principal disadvantage of this approach is that it is very time-consuming computationally, making it both expensive and inconvenient for use as an everyday design procedure.

The second category is inverse design methods, in which one specifies the surface pressure distribution and the method calculates the corresponding airfoil geometry. This approach was pioneered by Lighthill[3] for incompressible flow using a conformal mapping technique. For transonic flows, the two principal approaches are potential methods and hodograph methods. The potential methods, such as the GRUMFOIL code of Volpe and Melnik,[4] solve the nonlinear, isentropic, full-potential equations in the physical plane. The hodograph methods, of which the method by Bauer et al.[5] was the first and remains the most widely used, solve the full-potential equations in the hodograph plane in which the equations are linear. Both methods have been extended to include viscous effects through a boundary-layer displacement thickness calculated using an integral boundary-layer analysis. The potential methods are more flexible because they also have the capability to be used in a direct, analysis mode so that, after designing a new airfoil, the same method can then analyze its off-design performance. A limitation of both approaches is the assumption of isentropic flow, which leads to an incorrect treatment of shocks. The advantage of these methods over the optimization methods previously described is that they are much faster, allowing interactive design at workstations.

This paper presents results obtained using a new analysis/design code, ISES, which has been developed over the last three years.[6-9] In concept, it has similarities to both the streamline curvature analysis methods used in turbomachinery calculations and the full potential inverse design methods. Its origins lie in research on solving the Euler equations for quasi-one-dimensional streamtubes.[10,11] The current method is the natural two-dimensional extension of this work, which treats the two-dimensional flow as a set of streamtubes coupled through the position of, and pressure at, the streamline interfaces. The unknown variables are both the velocity of the fluid and the position of the streamlines. The discrete Euler equations are assembled as a system of nonlinear equations and solved simultaneously using the Newton method. Unlike the full-potential methods, this method solves the Euler equations in conservative integral form and so correctly handles shocks. As with the potential methods at the airfoil surface, one can either specify the position of the surface streamline (direct, analysis

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