Journal of Propulsion and Power
Vol. 23, No. 5, September–October 2007

Three-Dimensional Nonreflecting Boundary Conditions for Swirling Flow in Turbomachinery

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DOI: 10.2514/1.22117

This paper discusses the implementation of nonreflecting boundary conditions for the computation of linear unsteady aerodynamic turbomachinery problems. Based on the use of precalculated far-field acoustic eigenmodes for a mean flow that is assumed to be uniform axially and circumferentially, but nonuniform in the radial direction, the method very effectively reduces the reflections and improves the convergence rate for both inviscid and viscous flows. Extension of the implementation within a generalized minimal residual method is summarized and convergence results are presented. This is the companion paper of a previous publication that addressed the numerical computation of the eigenmodes and their use for postprocessing.

I. Introduction

THERE are different approaches to analyze turbomachinery unsteadiness. These methods vary from the use of linearized flow solvers based on the observation that the unsteadiness is sufficiently small to be considered a linear perturbation to a steady flow, to fully nonlinear 3-D unsteady methods. Among all of these, the linear harmonic Euler and Navier–Stokes methods have become very popular in industry, due to a successful compromise between accuracy and computational cost, and constitute the background to this paper.

Whatever the method used and the problem under investigation, the numerical solution is calculated on a truncated finite domain, and one must prevent any nonphysical reflections of outgoing waves at the far-field boundaries that could contaminate the numerical solution. This becomes essential in turbomachinery applications in which the boundaries are often not very far from the blades, because the physical spacing between the blade rows can be quite small. It therefore becomes highly important for an accurate simulation to construct nonreflecting boundary conditions (NRBCs).

Preventing spurious reflections that would corrupt the solution is not only important to get an accurate prediction of the flowfield, but also to get more efficient computations; convergence rate is enhanced due to an improvement of the transmission of outgoing waves, allowing smaller meshes to be used.

There is already a very broad and diverse existing theory for different applications. In computational fluid dynamics (CFD), the most common techniques employed use an analytical approach to describe the eigenmodes of the governing equations. In 1975, Adamczyk et al. [1] constructed exact NRBCs for the potential equation when calculating linearized unsteady flows by matching the known analytical solution with the computed one. Extended by Hall and Crawley [2] to the linearized Euler equations, Giles [3], in 1990, introduced exact nonreflecting 2-D boundary conditions based on the ideas of Engquist and Majda [4], using a characteristic analysis of the linearized equations. Later, Saxer and Giles [5] extended the steady NRBCs into a quasi-3-D formulation, assuming the circumferential variation is much larger than the radial variation, and successfully applied it to the solution of axial turbine stages. Finally, in 1996, Fan and Lakshminarayana [6] followed the same approach for unsteady flows and demonstrated their effectiveness for turbine calculations. In the field of computational aeroacoustics, people have developed other approaches based either on the discrete system-building numerical NRBCs (e.g., Rowley and Colonius [7]) or on asymptotic expansions (e.g., Grote and Keller [8] and Tam [9]).

In this paper, we are interested in solving complex turbomachinery problems in which the radial variations in the mean and unsteady perturbation flowfields cannot be ignored. For that purpose, we intend to resume a theory described by Lorence et al. [10], who generalized Giles's [3] approach to the 3-D Euler equations using a mixed analytical and numerical method to approximate the 3-D eigenmodes. In their paper, Hall et al. [10] demonstrated the effectiveness of their method to eliminate the reflections at the boundaries through the visualization of pressure contours that were shown to pass smoothly out of the computational domain. The test case simulated the flow through a cascade of cambered airfoil and is known as the tenth standard configuration [11]. The present study addresses the extension of the approach to the 3-D Navier–Stokes equations.

The primary objective is to determine the appropriate acoustic eigenmodes for a swirling axisymmetric mean flow using numerical eigenvalue and eigenvector computations and use them throughout the calculation to enforce NRBCs. The first key step is to compute a numerical approximation to the eigenmodes for linear unsteady flow perturbations superimposed upon an inviscid/viscous mean flow, which is swirling and axisymmetric.

To achieve this, a preprocessor tool solving the linear 3-D cylindrical Euler/Navier–Stokes equations was developed using LAPACK, a general-purpose numerical linear algebra library.§ Because of the axisymmetry of the mean flow, the eigenmodes are Fourier modes circumferentially. Considering each Fourier mode separately and discretizing the relevant ordinary differential equation in the radial direction, the eigenvalues of the resulting matrix give the complex axial wave number, and the corresponding right and left eigenvectors give the eigenmodes of the partial differential equation. The complex axial wave number is used to decompose the flowfield into upstream and downstream traveling eigenmodes, whereas the

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§Data available online at http://www.netlib.org/lapack [retrieved 24 July 2007].