Aerodynamic optimisation for complex geometries

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Overview

- grid generation
- sensitivity analysis
- adjoint approach
- applications

Grid Generation

For design purposes we need to be able to create

- a base grid for given values of design parameters
- perturbed grids for perturbed values

The linear/nonlinear perturbed grids have the same topology as the base grid, so any objective function varies smoothly.

Grid Generation

Base grid generation:

- parametric solids-based EPD system defines solid surface as a collection of surface patches separated by lines terminated by points
- surface is gridded in order of increasing dimensionality (point, line, surface patch)
- interior grid nodes are then created by advancing front or Delauney algorithms

Grid Generation

Perturbed grid generation:

- parametric perturbation defines perturbation to solid surfaces and separating lines
- surface grid node perturbations are defined in order of increasing dimensionality (point, line, surface patch)
- interior grid nodes are perturbed using 'method of springs' or elliptic p.d.e.

Grid Generation

Method of springs:

- edges of grid are modelled as springs
- base grid is defined to be in equilibrium
- perturbation to surface points disturbs equilibrium, leading to perturbation to interior nodes to re-establish it
- strength of springs is defined to ensure no cross-over in the boundary layer
- (similar idea can be used to define surface perturbations in the first place)

Grid Generation

In the elliptic p.d.e. approach, grid node perturbations $\tilde{\boldsymbol{x}}(\boldsymbol{x})$ are defined by

$$\nabla \cdot (k(\boldsymbol{x}) \nabla \tilde{\boldsymbol{x}}) = 0,$$

subject to specified boundary conditions.

$k(\boldsymbol{x})$ is defined to ensure no cross-over in boundary layers.

Nonlinear Sensitivity

For a single design variable α, discrete flow equations

$$\boldsymbol{F}(\boldsymbol{U}, \alpha) = 0,$$

define flow field $\boldsymbol{U}$ as a function of α.

Gradient of objective function $I(\boldsymbol{U}, \alpha)$ can be approximated by

$$\frac{dI}{d\alpha} \approx \frac{I(\boldsymbol{U}(\alpha+\epsilon), \alpha+\epsilon) - I(\boldsymbol{U}(\alpha), \alpha)}{\epsilon}.$$

Easily generalised to multiple design variables, at cost of extra calculations.

Linear Sensitivity

Linearising discrete flow equations gives

$$\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{U}} \, \widetilde{\boldsymbol{U}} + \frac{\partial \boldsymbol{F}}{\partial \alpha} = 0,$$

where

$$\frac{\partial \boldsymbol{F}}{\partial \alpha} \equiv \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{X}} \, \frac{\partial \boldsymbol{X}}{\partial \alpha}.$$

i.e. change in α perturbs grid coordinates which perturb flux residuals.

$\widetilde{\boldsymbol{U}}$ represents flow perturbation as seen by perturbed grid point

Linear Sensitivity

Gradient of objective function is given by

$$\frac{dI}{d\alpha} = \frac{\partial I}{\partial \boldsymbol{U}} \, \widetilde{\boldsymbol{U}} + \frac{\partial I}{\partial \alpha}.$$

Generalisation to multiple design parameters requires separate calculation for each, so no particular benefit compared to nonlinear sensitivities.

Discrete Adjoint

Substituting for $\widetilde{\boldsymbol{U}}$ gives

$$\frac{dI}{d\alpha} = -\frac{\partial I}{\partial \boldsymbol{U}} \left(\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{U}} \right)^{-1} \frac{\partial \boldsymbol{F}}{\partial \alpha} + \frac{\partial I}{\partial \alpha},$$

which can be written as

$$\frac{dI}{d\alpha} = \boldsymbol{V}^T \frac{\partial \boldsymbol{F}}{\partial \alpha} + \frac{\partial I}{\partial \alpha},$$

where $\boldsymbol{V}$ satisfies the adjoint equation

$$\left(\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{U}} \right)^T \boldsymbol{V} + \left(\frac{\partial I}{\partial \boldsymbol{U}} \right)^T = 0.$$

Discrete Adjoint

The advantage of the adjoint approach is that the same adjoint solution $\boldsymbol{V}$ can be used for each design variable, since $\boldsymbol{V}$ depends on I but not α.

The drawback is that because $\boldsymbol{V}$ depends on I a separate calculation must be performed for each constraint function.

Question: in real engineering applications, how many design variables and constraints are there?

Analytic Adjoint

The analytic adjoint is more complicated. Critical first step is formulation of linear perturbation equations.

Simple linearisation of 2D Euler equations

$$\frac{\partial}{\partial x} F_x(U) + \frac{\partial}{\partial y} F_y(U) = 0,$$

yields

$$\frac{\partial}{\partial x}(A_x \widetilde{U}) + \frac{\partial}{\partial y}(A_y \widetilde{U}) = 0,$$

where $\widetilde{U}$ is perturbation <u>at a fixed point</u>.

Analytic Adjoint

However, linearising the b.c.

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0,$$

gives

$$\tilde{\boldsymbol{u}} \cdot \boldsymbol{n} + (\tilde{\boldsymbol{x}} \cdot \nabla \boldsymbol{u}) \cdot \boldsymbol{n} + \boldsymbol{u} \cdot \tilde{\boldsymbol{n}} = 0,$$

which is hard to discretise accurately.

This is similar to discrete adjoint treatment with no perturbation to interior grid points.

Analytic Adjoint

Start instead with generalised coordinates,

$$\frac{\partial}{\partial \xi}\left(F_x \frac{\partial y}{\partial \eta} - F_y \frac{\partial x}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(F_y \frac{\partial x}{\partial \xi} - F_x \frac{\partial y}{\partial \xi}\right) = 0.$$

Now define perturbed coordinates as

$$x = \xi + \alpha X(\xi, \eta), \qquad y = \eta + \alpha Y(\xi, \eta),$$

where $X(\xi, \eta)$ and $Y(\xi, \eta)$ are smooth functions which match the surface perturbations due to the design variable α.

Analytic Adjoint

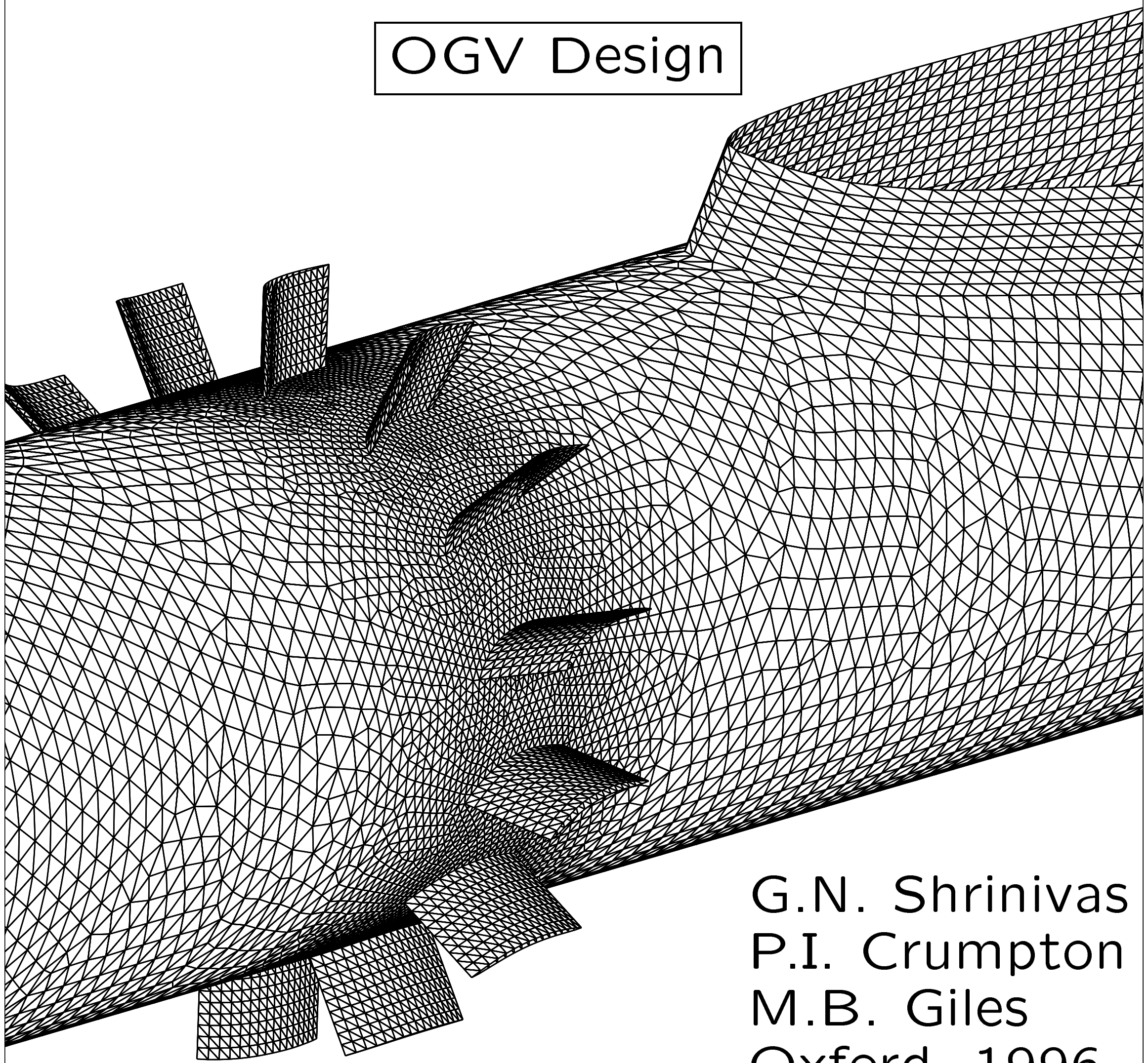
Linearising with respect to α yields

$$\begin{aligned}\frac{\partial}{\partial\xi}(A_x\widetilde{U})+\frac{\partial}{\partial\eta}(A_y\widetilde{U}) \;=\; & -\frac{\partial}{\partial\xi}\left(F_x\frac{\partial Y}{\partial\eta}-F_y\frac{\partial X}{\partial\eta}\right)\\ & -\frac{\partial}{\partial\eta}\left(F_y\frac{\partial X}{\partial\xi}-F_x\frac{\partial Y}{\partial\xi}\right),\end{aligned}$$

where $\widetilde{U}$ is now the perturbation in the flow variables for fixed (ξ,η) rather than fixed (x,y).

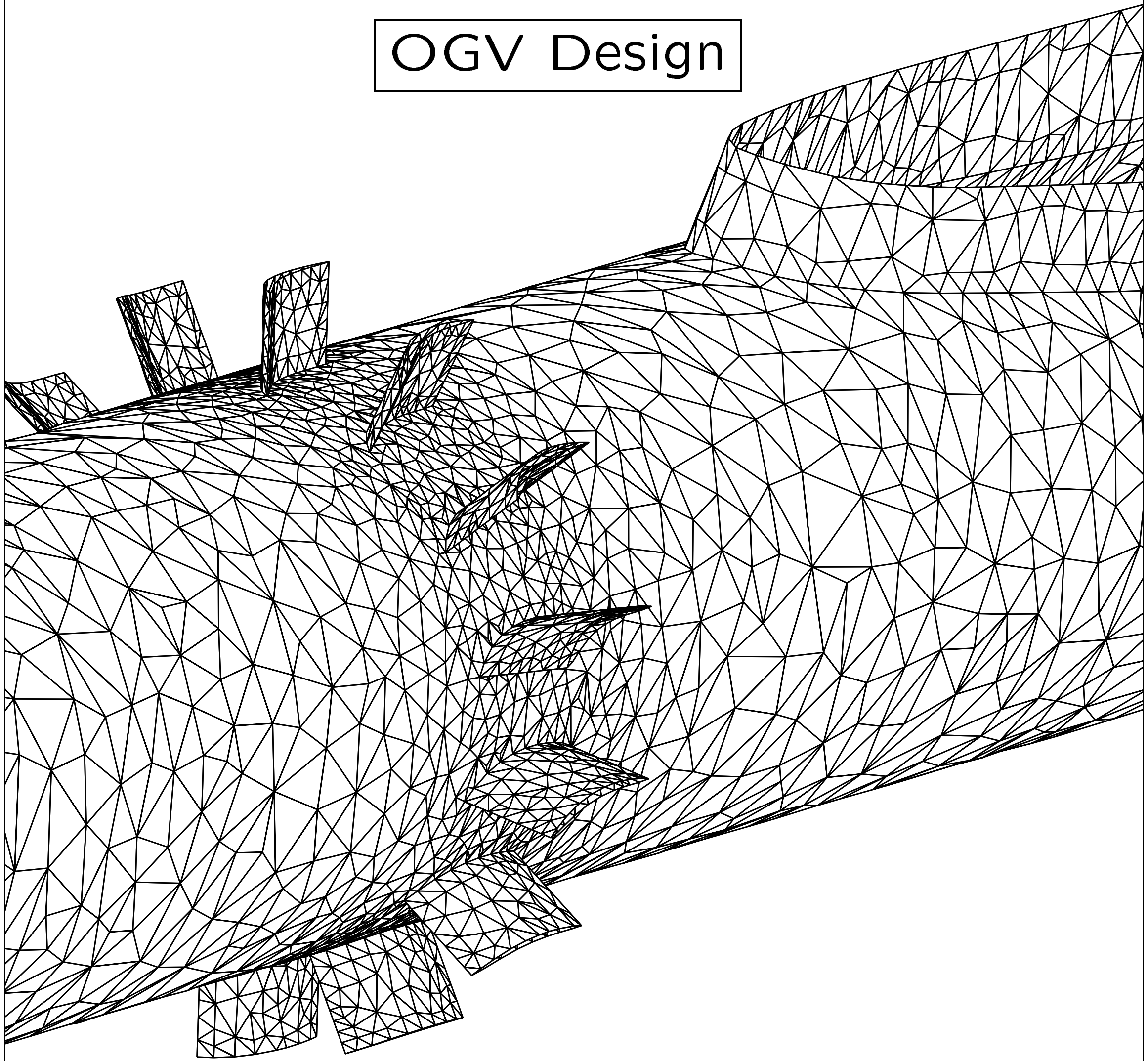
The linearisation of the b.c.'s is simple, and the overall accuracy is much better.

OGV Design

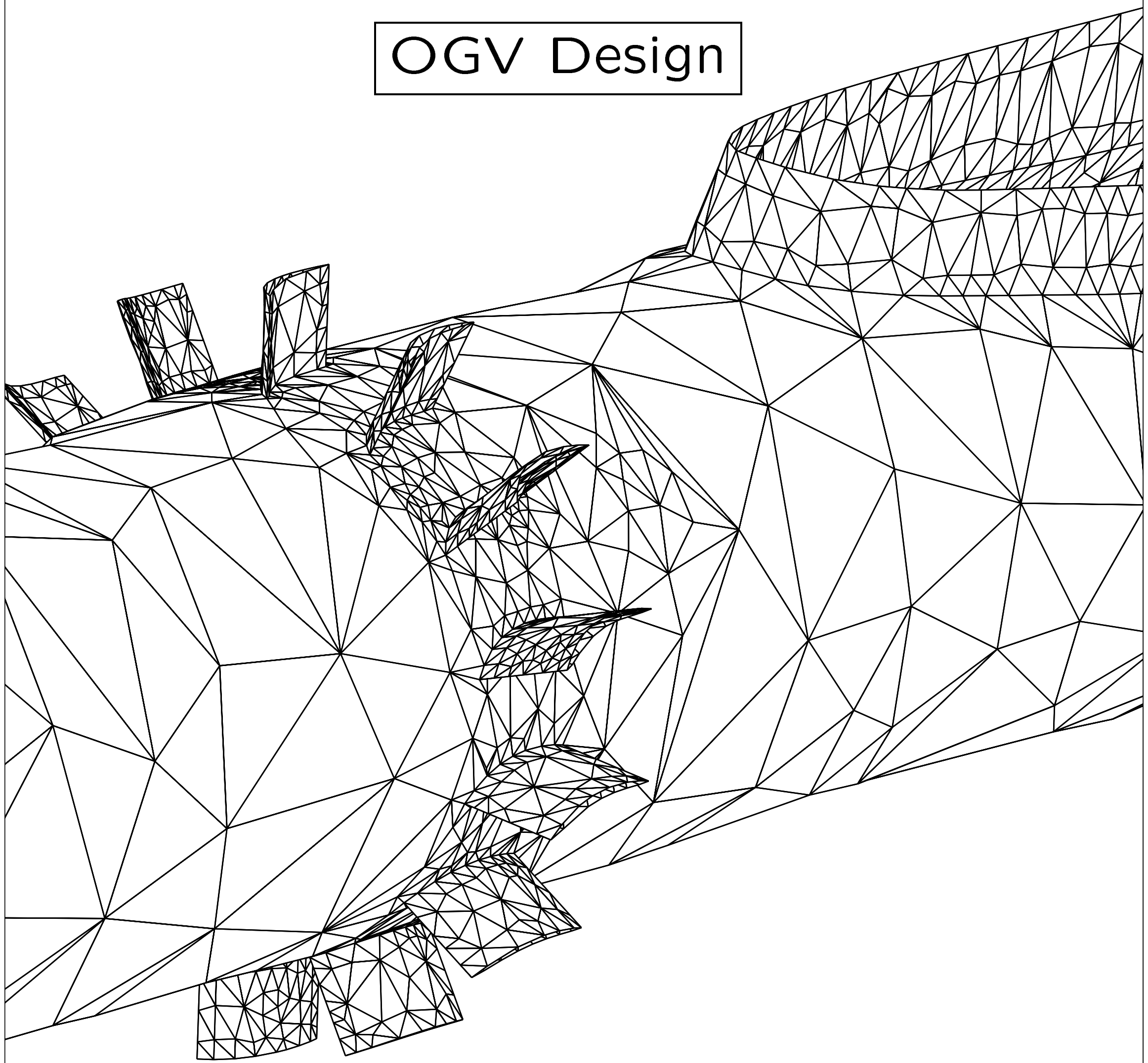


G.N. Shrinivas
P.I. Crumpton
M.B. Giles
Oxford, 1996

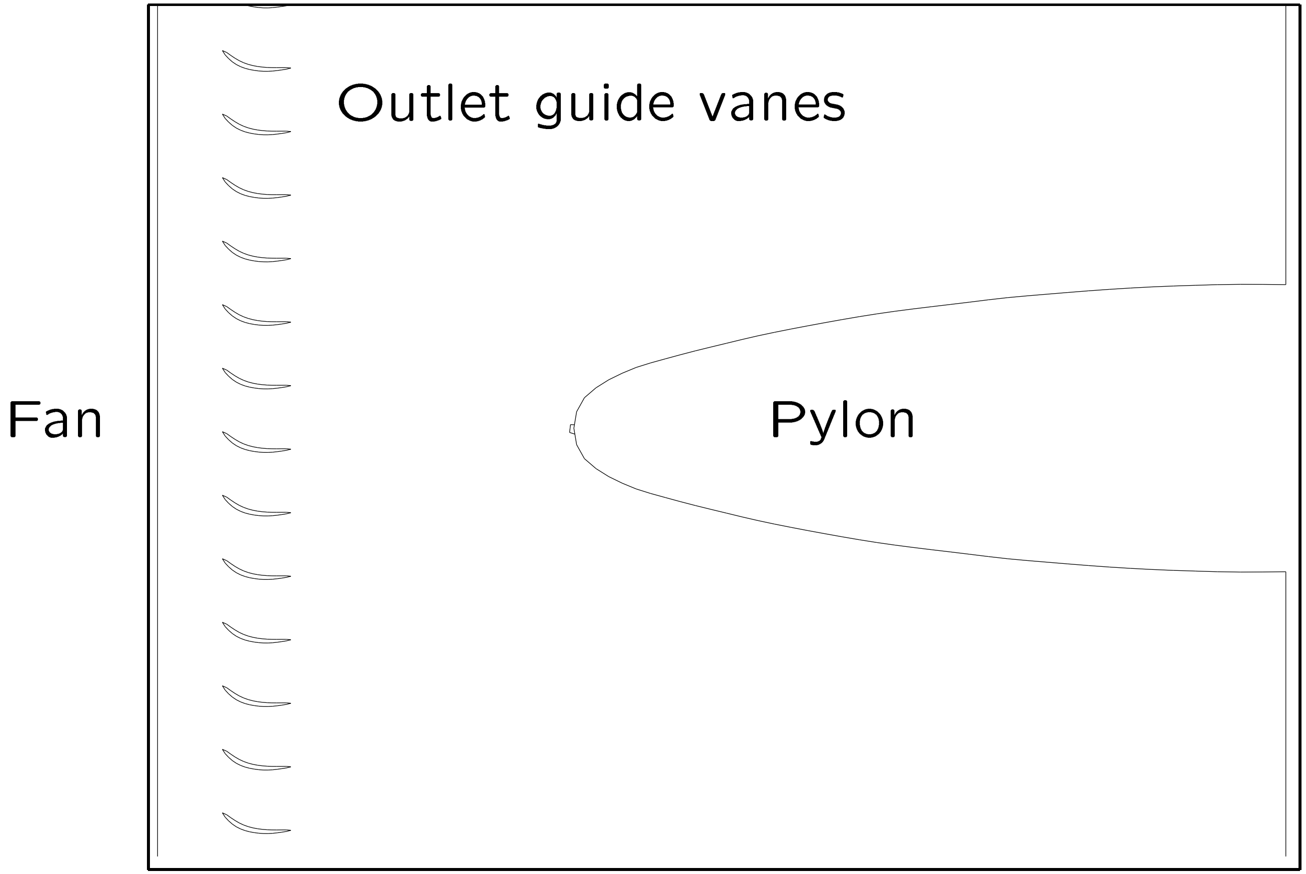
OGV Design



OGV Design



OGV Design



OGV Design

Objective is to minimise circumferential pressure variation upstream of the OGV's by changing their camber.

Optimisation uses

- unstructured grid with 560k tetrahedra
- Euler equations
- multigrid and parallel computing
- elliptic p.d.e. for grid perturbation
- nonlinear sensitivities and quasi-Newton optimisation

OGV Design

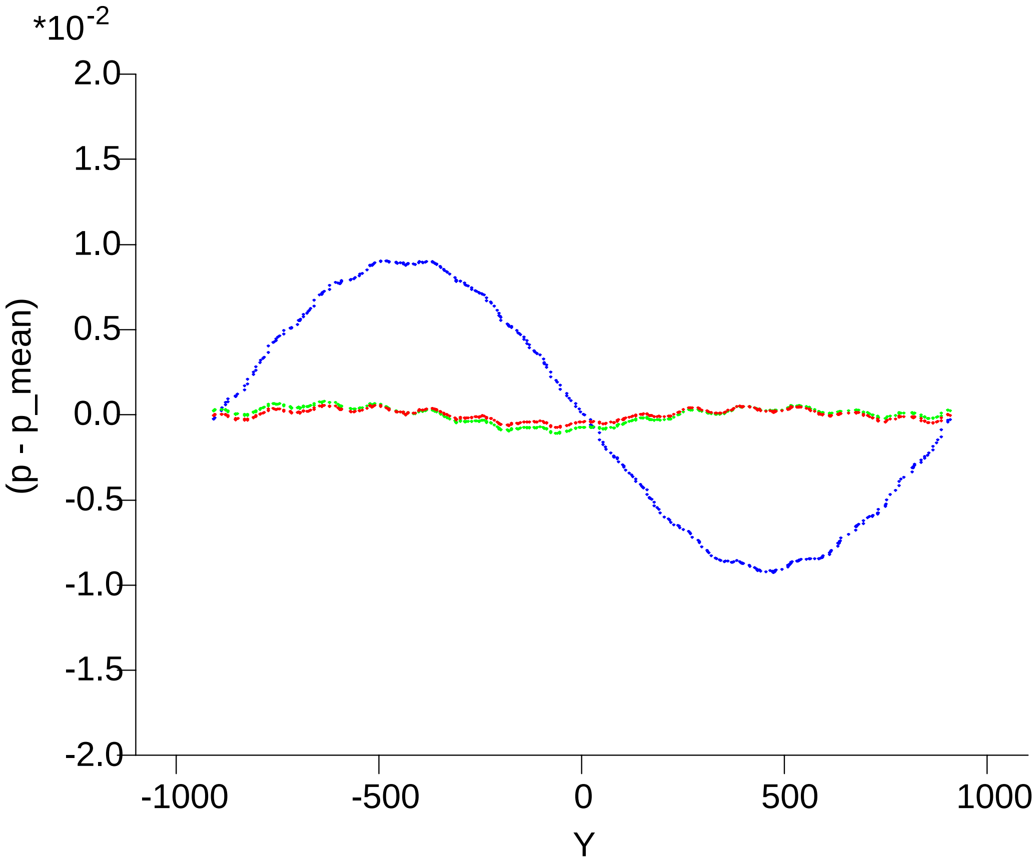
Leading edge of each OGV is left unchanged due to uniform flow incidence; camber change varies linearly with distance from leading edge to change the outflow angle.

First design exercise uses a camber change which varies sinusoidally with circumferential angle.

Only 2 design variables: maximum change at hub and tip

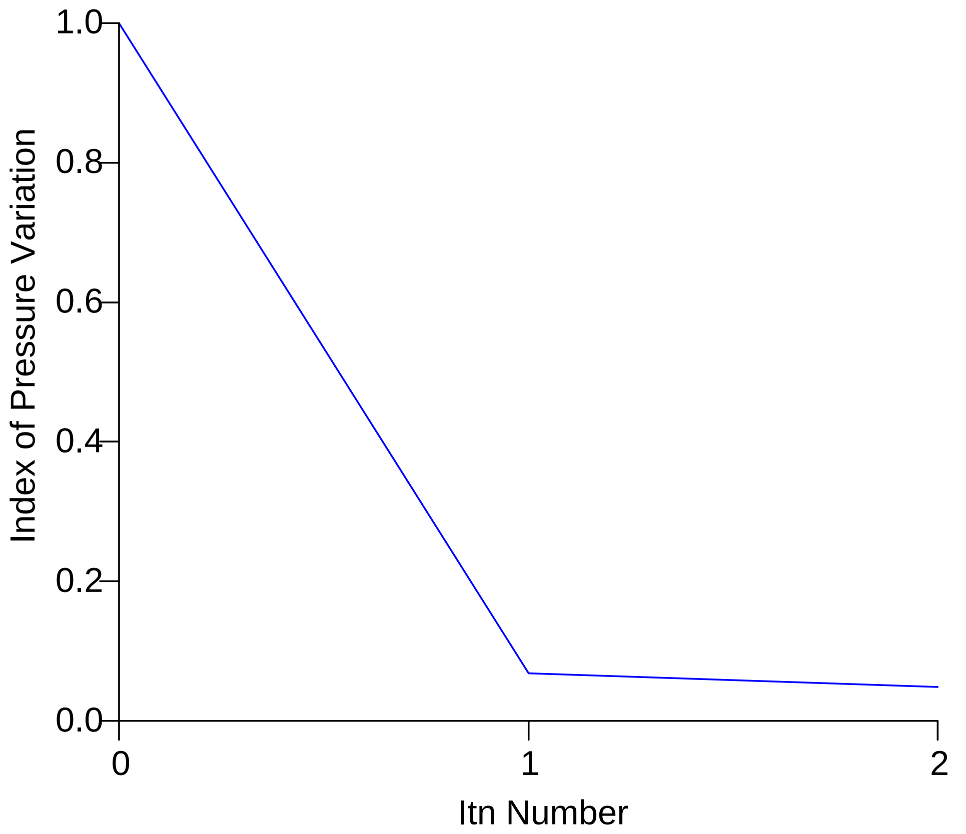
OGV Design

Optimisation using sinusoidal variation



OGV Design

Optimisation using sinusoidal variation



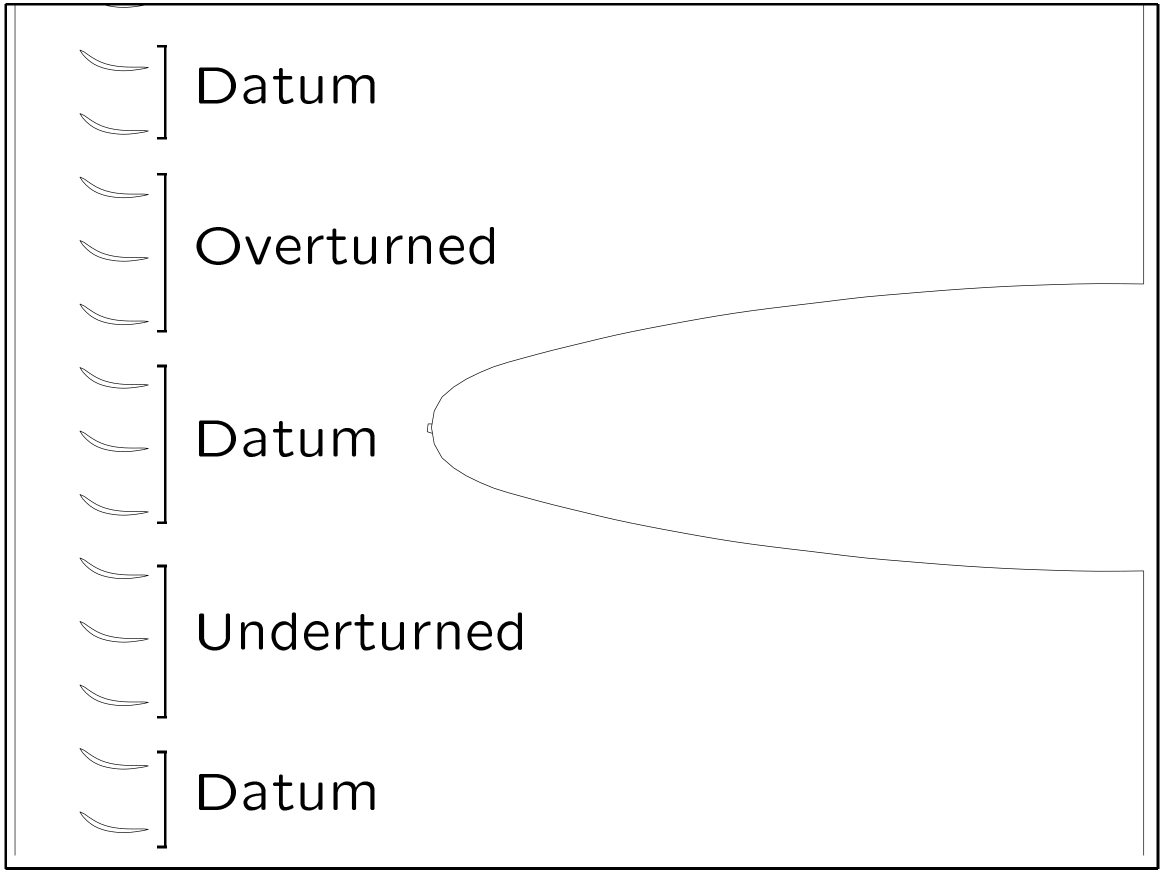
OGV Design

Drawback of this design is that all OGV's are different.

Second design exercise uses just 3 blade types, the original, one with overturning and one with an equal amount of underturning.

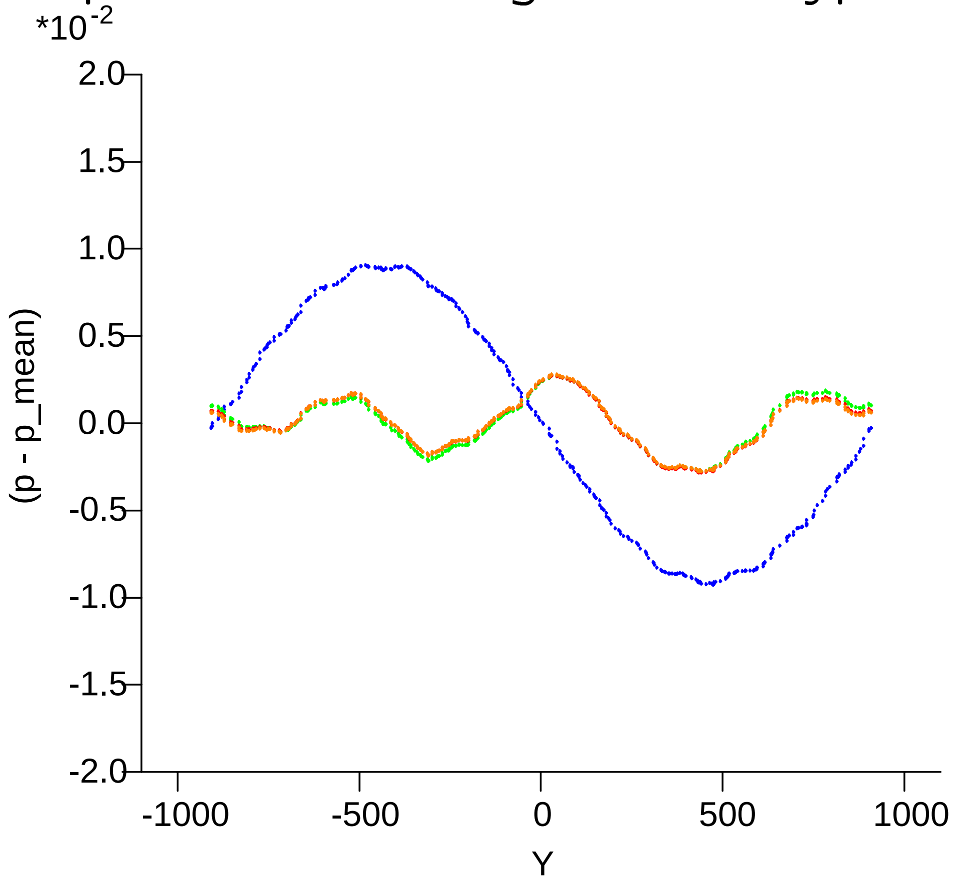
Still only 2 design variables: maximum change at hub and tip

OGV Design



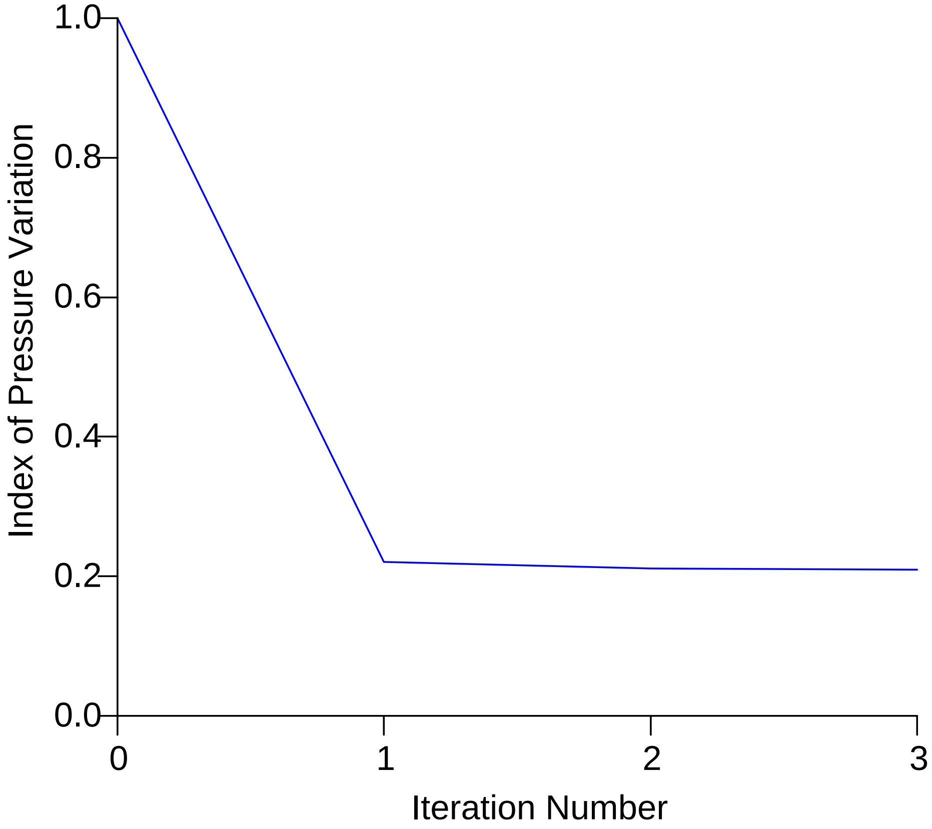
OGV Design

Optimisation using 3 blade types



OGV Design

Optimisation using 3 blade types



Business Jet

J. Elliott and J. Peraire, MIT, 1996

Business Jet

Objective function is mean-square deviation from a target pressure distribution for a ‘clean’ wing in the absence of the rear nacelle.

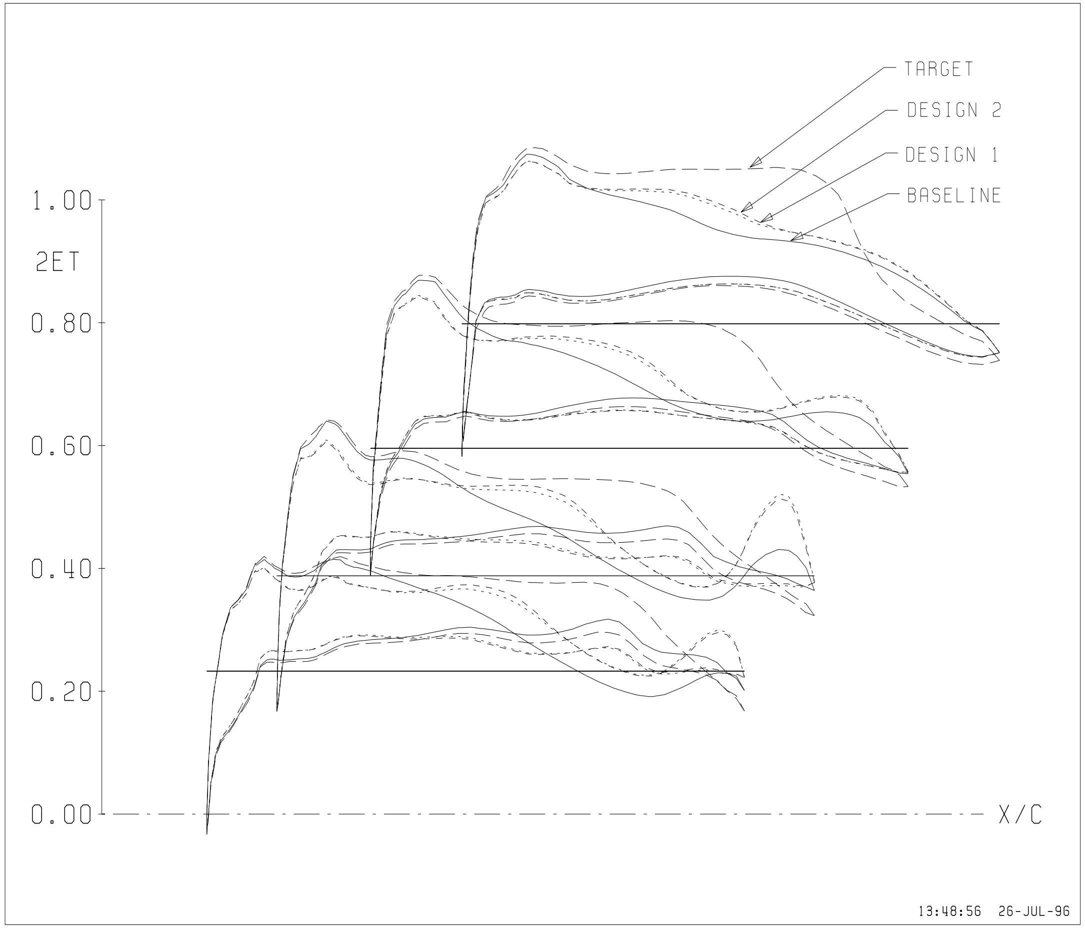
6 design variables are used to define smooth perturbations to the wing.

Business Jet

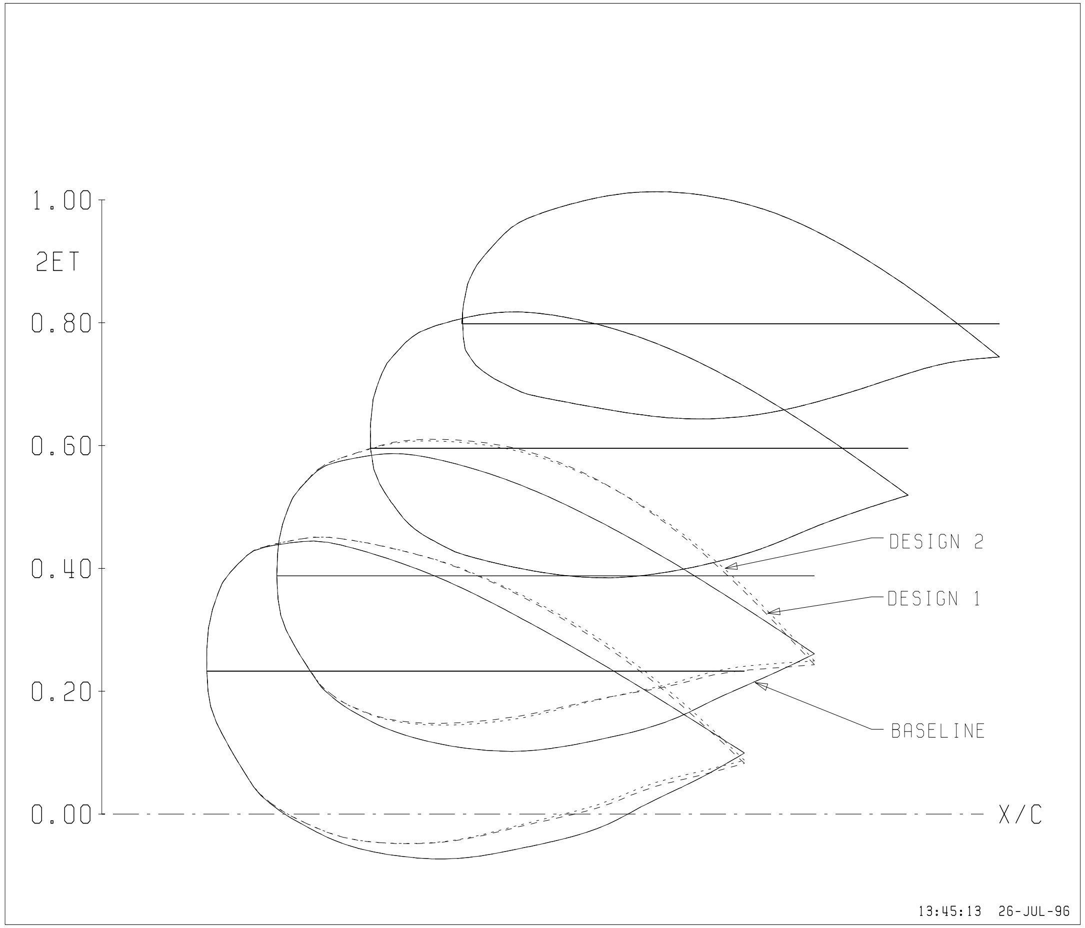
Optimisation uses

- unstructured grid with 860k tetrahedra
- Euler equations
- multigrid and parallel computing
- method of springs for grid perturbation
- BFGS optimisation method with discrete adjoint formulation for gradients

Business Jet



Business Jet



Conclusions/Future

- aerodynamic optimisation for complex geometries is becoming a reality
- with multigrid and parallel computing, costs are now acceptable for inviscid modelling; viscous modelling is under development but will cost up to 5 times as much
- grid generation for base grids and perturbed grids is a critical component
- pros and cons of different optimisation methods has yet to be properly investigated