Numerical Methods II
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Practical 2

1. For the case of Geometric Brownian Motion and a European call option, with parameters, $r=0.05$, $\sigma=0.2$, $T=1$, $S(0)=100$, $K=100$, investigate the following forms of variance reduction:

   (a) First, try antithetic variables using $\frac{1}{2}\left(f(W)+f(-W)\right)$ where W is the value of the underlying Brownian motion at maturity.

   What is the estimated correlation between $f(W)$ and $f(-W)$? How much variance reduction does this give?

   (b) Second, try using $\exp(-rT)\, S(T)$ as a control variate, noting that its expected value is $S(0)$.

   Again, how much variance reduction does this give?

2. For the case of Geometric Brownian Motion and a digital put option, with parameters, $r=0.05$, $\sigma=0.2$, $T=1$, $S(0)=100$, $K=50$, investigate the use of importance sampling:

   (a) First, estimate the value without importance sampling.

   How many samples are needed to obtain a value which is correct to within 10%? (i.e. the 3 standard deviation confidence limit corresponds to $\pm$ 10%).

   (b) Second, try using importance sampling, adjusting the drift so that half of the samples are below the strike at maturity, and the other half are above.

   Now how many samples are required to get the value correct to within 10%?

3. For the same application as question 1, use the finite difference "bumping" method to compute delta and vega, the sensitivities to changes in the initial price and the volatility.

   Check your results are correct by comparing to the analytic values given by `european_call`.

   Experiment with and without using the same random numbers for the two sets of samples to see the effect on the variance.

   Also experiment with the size of the "bump" to see the effect on the accuracy.