Module 6: Monte Carlo question (Mike Giles)

The aim in this question is to investigate the performance of the Quasi-Monte Carlo method using Sobol points, with digital scrambling for the randomisation.

The test cases are all basket options on 5 underlying assets, each modelled as Geometric Brownian Motion, and you should use an Euler-Maruyama discretisation.

Parameter values:

- risk-free interest rate $r = 0.02$
- volatility $\sigma_i = 0.2$ for all 5 assets
- correlation $\rho_{ij} = 0.2, i \neq j$
- initial asset prices $S_i(0) = 100$ for all 5 assets
- maturity $T = 1$
- 64 timesteps, so $\Delta t = T/64$
- a total of 2^{15} paths, split into 32 groups, each with its own randomisation, as described in the lecture

Options on $\overline{S}$, the arithmetic average of 5 assets:

- call option, $P = \exp(-rT)\,(\overline{S}(T) - K)^+$,
- Asian option, $P = \exp(-rT)\,(T^{-1}\int_0^T \overline{S}(t)\,\mathrm{d}t - K)^+$
- lookback option, $P = \exp(-rT)\,(\max_t \overline{S}(t) - K)^+$
- strike $K = 100$

For each option, experiment with the following:

- Brownian Bridge or Cholesky factorisation of the covariance matrix in time
- PCA or Cholesky factorisation of the correlation matrix for the multiple assets

and also compare the performance to standard Monte Carlo.

Important: you may find it helpful to look at the following MATLAB codes which are available at `http://people.maths.ox.ac.uk/gilesm/mc/`

- `lec5c.m` – demonstrates the use of Sobol routines in MATLAB for an option on a basket of 5 assets, but without any time discretisation (also shows effectiveness of Latin Hypercube sampling)
- `qmc.m` – demonstrates the effectiveness of QMC sampling using rank-1 lattice points for an option on a single underlying asset, with time discretisation
- `bb.m` – a routine used by `qmc.m` which implements the Brownian Bridge construction

You should hand in the code (either in MATLAB, or C/C++ if you have access to a numerical library which generates Sobol points) and a short writeup, presenting your results as a table for each option of

- values (the average of the averages produced by the 32 randomisations)
- standard deviations (the estimated standard deviation of that average of averages)

Comment on whether the results are as expected.