Module 6: Monte Carlo question (Mike Giles)

The aim in this question is to investigate the performance of the Quasi-Monte Carlo method using Sobol points, with digital scrambling for the randomisation.

The test case is Heston's stochastic volatility model:

$$\begin{aligned} \mathrm{d}S &= r\,S\,\mathrm{d}t + \sqrt{\nu}\,S\,\mathrm{d}W_1 \\ \mathrm{d}\nu &= \lambda\,(\sigma^2 - \nu)\,\mathrm{d}t + \xi\,\sqrt{\nu}\,\mathrm{d}W_2 \end{aligned}$$

with correlation ρ between $\mathrm{d}W_1$ and $\mathrm{d}W_2$, which you should approximate using the following Euler-Maruyama discretisation which has a slight modification to avoid problems if ν becomes negative:

$$\begin{aligned} \widehat{S}_{n+1} &= \widehat{S}_n + r\,\widehat{S}_n\,\Delta t + \sqrt{|\widehat{\nu}_n|}\,\widehat{S}_n\,\Delta W_1 \\ \widehat{\nu}_{n+1} &= \widehat{\nu}_n + \lambda\,(\sigma^2 - \widehat{\nu}_n)\,\Delta t + \xi\,\sqrt{|\widehat{\nu}_n|}\,\Delta W_2 \end{aligned}$$

Parameter values:

- risk-free interest rate $r = 0.02$
- long-term volatility $\sigma = 0.2$
- vol-of-vol $\xi = 0.2$
- mean reversion rate $\lambda = 2$
- correlation $\rho = -0.2$
- initial data $S(0) = 100$, $\nu(0) = 0.04$
- maturity $T = 1$, and 64 timesteps, so $\Delta t = T/64$
- a total of 2^{15} paths, split into 32 groups, each with its own randomisation, as described in the lecture

Calculate the value of these two options:

- call option, $P = \exp(-rT)\ (S(T) - K)^+$,
- barrier option, $P = \exp(-rT)\ (S(T) - K)^+\ \mathbf{1}_{\inf_{[0,T]} S_t > B}$,
- strike $K = 100$, barrier $B = 80$

For each option, experiment with Brownian Bridge or Cholesky factorisation of the covariance matrix in time (as discussed in the lecture) and also compare the performance to standard Monte Carlo.

For the barrier option, use the approximation technique discussed in Module 4.

Important: you may find it helpful to look at the following MATLAB codes which are available at `http://people.maths.ox.ac.uk/gilesm/mc/`

- `lec5c.m` – demonstrates the use of Sobol routines in MATLAB for an option on a basket of 5 assets, but without any time discretisation (also shows effectiveness of Latin Hypercube sampling)
- `qmc.m` – demonstrates the effectiveness of QMC sampling using rank-1 lattice points for an option on a single underlying asset, with time discretisation
- `bb.m` – a routine used by `qmc.m` which implements the Brownian Bridge construction

You should hand in the code (either in MATLAB, or C/C++ if you have access to a numerical library which generates Sobol points) and a short writeup, presenting your results as a table of

- values (the average of the averages produced by the 32 randomisations)
- standard deviations (the estimated standard deviation of that average of averages)

Comment on whether the results are as expected.