

Problem Set 1: Probability

(Due Friday 2/17/2017 at 5pm)

Problem 1

Alice has a peculiar pair of four-sided dice. When she rolls the dice, the probability of any particular outcome is proportional to the product of values on each die. For example, if Alice rolls a 3 on the first die and a 4 on the second, then the probability is proportional to 3×4 . All outcomes that result in a particular product are equally likely.

- (a) What is the sample space? (show in a table as we did in lecture 1)
- (b) What is the probability of each outcome in the sample space? (hint: probabilities of all outcomes must sum to 1)
- (c) What is the probability of the product being even?
- (b) What is the probability of Alice rolling a 2 and a 3?

Problem 2

You have 100 emails in your inbox: 40 are spam, 60 are not. Of the 40 spam emails, 25 contain the sentence “one hundred percent free”. Of the 60 non-spam emails, only 2 contain “one hundred percent free”. If an email contains “one hundred percent free”, what is the probability that it is spam?

Problem 3

American Cancer Society estimates that about 2% of women have breast cancer. Susan G. Komen for The Cure Foundation states that “Overall, the sensitivity of mammography is about 84 percent. This means mammography correctly identifies about 84 percent of women who truly have breast cancer.” A recent study published in 2015 by the journal Cancer Epidemiology, Biomarkers & Prevention suggests that about 8% of all mammograms are false positive.

- (a) Prior to any testing and any information exchange between a patient and a doctor, what probability of having breast cancer should the doctor assign to the female patient?
- (b) If the patient goes through breast cancer screening and her mammogram yields a positive result, what is the probability that she has cancer?
- (c) Since the patient’s test resulted in a positive mammogram, what probability of having breast cancer should the doctor assign to her now?

(d) The doctor decides to do a second mammogram. Suppose that the second screening also resulted in a positive mammogram. Given the information in (c), what is the probability that she has cancer?

Problem 4

You are dealt **exactly** 2 cards from a well shuffled 52-card deck. The deck contains exactly 4 aces. What is the probability that both your cards are aces?

Problem 5

Mary and Tom go to the Angelika theatre to watch 20th Century Women, and they find an empty row that consists of 10 seats. Assume that each possible pair of seat locations is equally likely, and assume that Mary and Tom do not sit in the same chair. Calculate the probability that the seats they select are adjacent.