Game Theory: Lecture 1

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What is game theory?

- Strategic interactions between self-interested agents
- Requires: (i) at least two players; (ii) each player has a number of strategies that determine the outcomes; (iii) associated with each outcome is a numerical payoff.
- Games can include: traditional games (chess, poker); social and psychological games (voting, dating), biological games (predator-prey, evolution), economic games (financial or business choices)
- Notions of equilibrium
- We will discuss **pure strategy games** and **mixed strategy games**

TCP Backoff Game

TCP Backoff Game

Question: Should you send messages over the internet using correctly implemented TCP? Or is it better to use defective TCP (i.e. one that does not have the backoff mechanism)?

To answer this question, you have to understand how using each implementation will affect you, given that you have an idea about which implementation other users in the network will choose.

TCP Backoff Game

Let's consider a **two-player** game version of this, such that:

- If both players use a correct implementation: 1ms delay for each
- If one player uses a correct while the other uses a defective implementation: 4ms delay for the correct implementation, 0ms delay for the defective one
- If both use a defective implementation: 3ms delay for each

TCP Backoff Game

What actions should a player take?

What if the two players are allowed to communicate and cooperate?

Does it matter if your opponent is rational?

Self-Interested Agents and Utility Theory

- Agents have opinions/preferences
- Agents to do not want to harm others, they only want what is in their best interest
- Each agent has a utility function which quantifies the degree of preference across alternatives
- An agent acts to maximize the utility function
- Think of utility as a form of payoff

Representations of Games

Normal Form: list what payoffs players get as a function of their actions

Extensive Form: includes timing of moves, e.g. chess (white moves first)

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2x2 since we have 2 players

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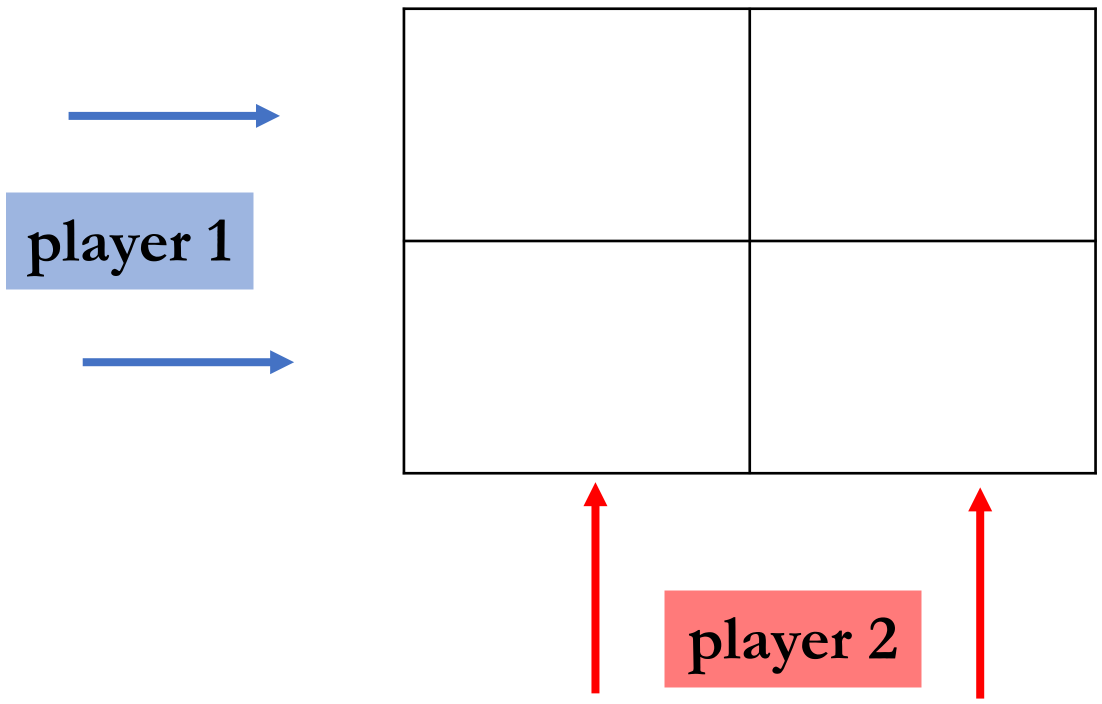

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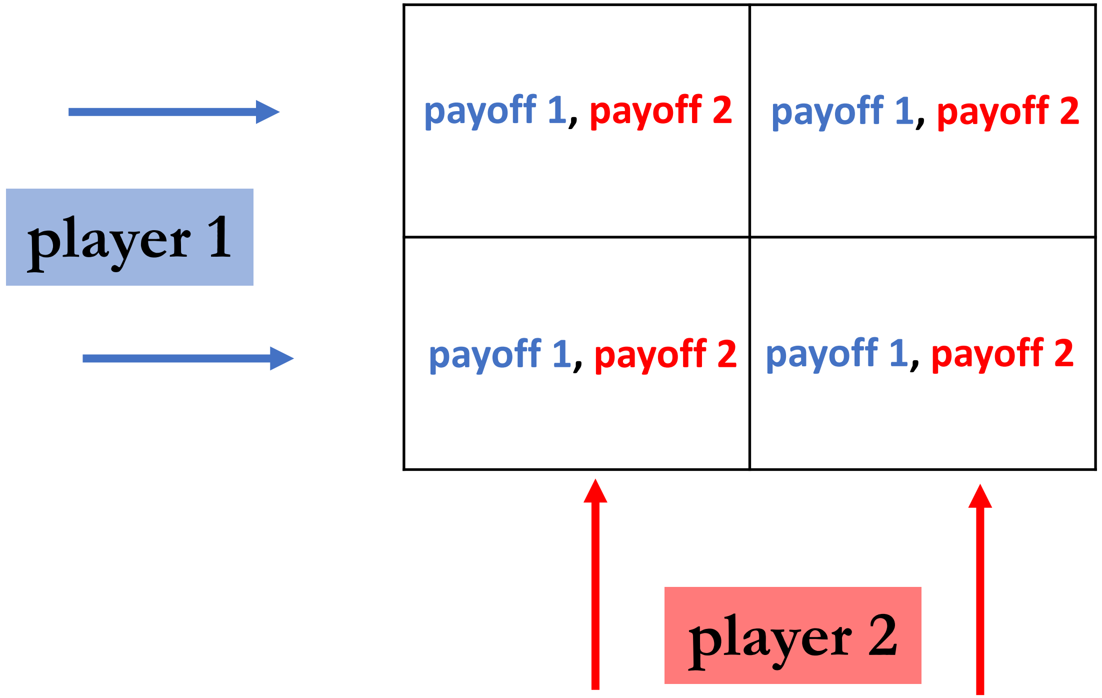

Representations of Games: TCP Backoff

The normal form game will be represented using a **matrix**, where:

- "row" player is player 1
- "column" player is player 2

	C	D
C	-1, -1	-4, 0
D	0, -4	-3, -3

player 1 (rows)

player 2 (columns)

Games of Pure Competition

- Players have exactly opposed interests
- Special case: zero-sum, where utility of player 1 is negative that of player 2

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Example: Matching pennies → one player wants to match, the other mismatch

player 1 \ player 2	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

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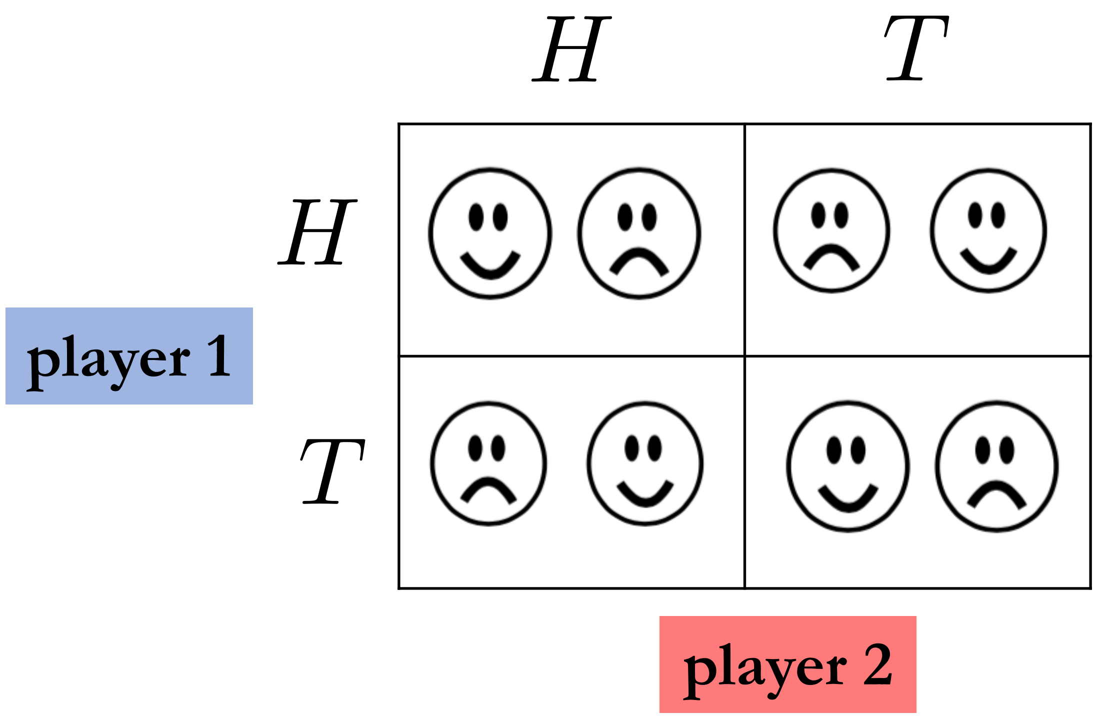
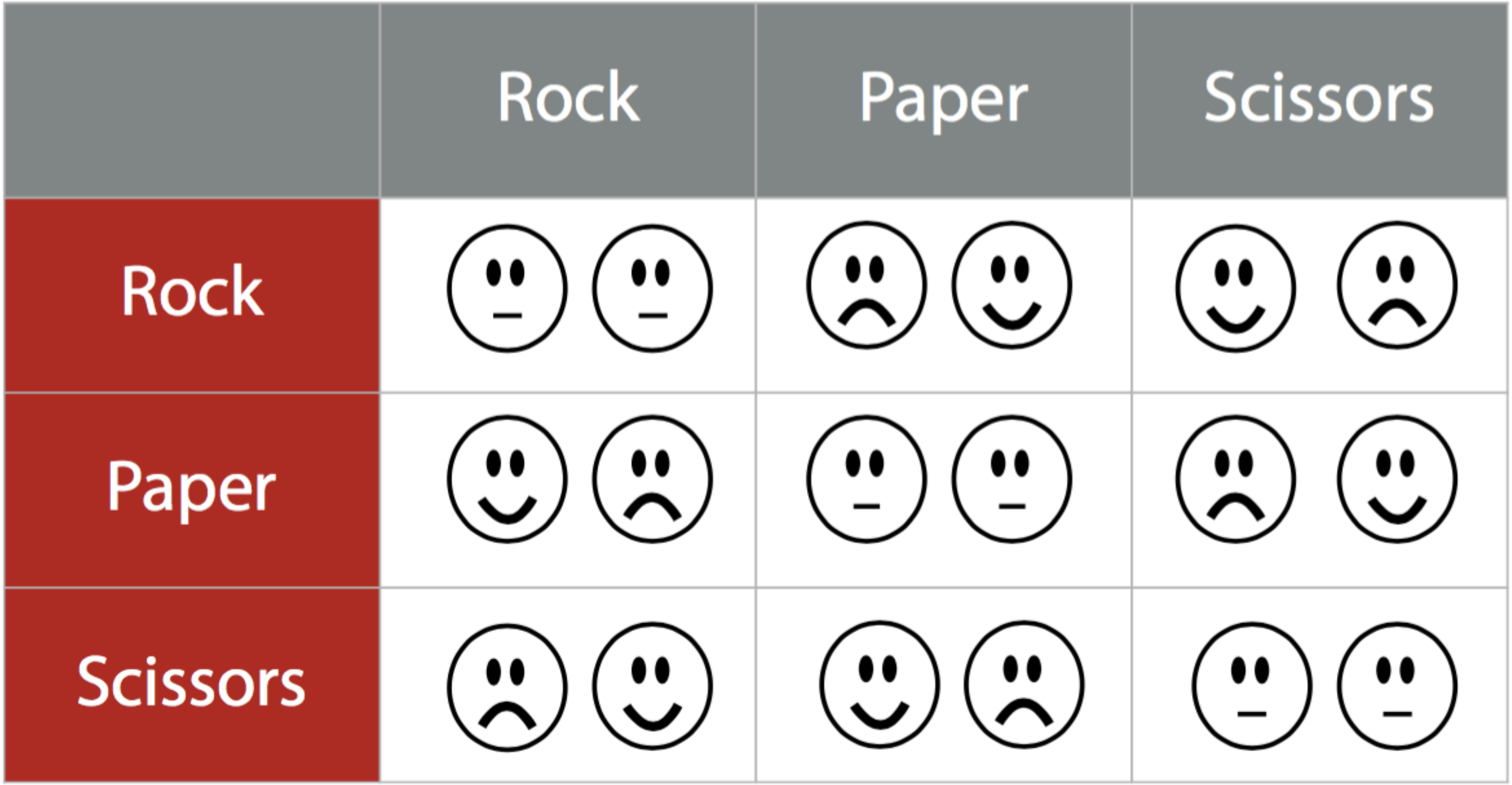


Games of Pure Competition: Rock-Paper-Scissors



Dominance Principle

COLIN

ROSE

	A	B	C	D
A	(12,-12)	(-1,1)	(1,-1)	(0,0)
B	(5,-5)	(1,-1)	(7,-7)	(-20,20)
C	(5,-5)	(2,-2)	(4,-4)	(3,-3)
D	(-16,16)	(0,0)	(0,0)	(16,0)

Begin with simple two-player zero-sum matrix game

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Begin with simple two-player zero-sum matrix game

Dominance Principle
A rational player should never play a dominated strategy

Dominance Principle

COLIN	A	B	C	D
ROSE A	(12,-12)	(-1,1)	(1,-1)	(0,0)
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- If Colin plays A, Rose should play A. If Rose plays A, then Colin should play B. ?

Dominance Principle

ROSE \ COLIN	A	B	C	D
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- If Colin plays A, Rose should play A. If Rose plays A, then Colin should play B … ?
- (CB) is a the most **cautious** strategy: an **equilibrium** solution
- Rose C is assured to win at least 2 (max in column)
- Colin B is assured to lose no more than 2 (min in row)

Dominance Principle

COLIN

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Definition: saddle point
An outcome is a saddle point if it is a minimum in its row and a maximum in its column

Saddle point principle
If a matrix game has a saddle point, both players should play it

Games of Cooperation

- Players have exactly the same interests
- No conflict

Example: Which side of the road should you drive on?

	Left	Right
Left		
Right		

Games of Cooperation

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Example: Which side of the road should you drive on?

	Left	Right
Left	☺ ☺	☹ ☹
Right	☹ ☹	☺ ☺

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Example: Battle of the sexes?

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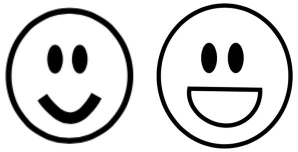
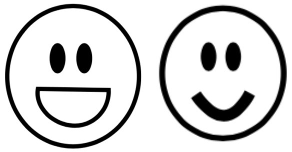
Movie		
Play		

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Example: Battle of the sexes?

	Movie	**Play**
Movie		
Play		

Keynes' Beauty Contest Game

- You hold a certain stock
- The stock price is rising
- You think the price is unjustifiably high
- You want to sell, but you would like to wait until the price is at its peak
- So you want to get out of the market **just** before other investors do
- What's your strategy?

Keynes’ Beauty Contest Game

- There are many players
- Each player chooses an integer between 1 and 100
- The player who names the integer closest to 2/3 of the average wins a prize

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How do you solve this?

Think strategically: what will other players do?

How do I respond?

Choose your optimal response, i.e. the one that will give you the highest payoff. This response will be called a **Nash Equilibrium**.

Nash Equilibrium

- List of actions + list of payoffs for player to choose from
- Players choose actions that maximize their payoffs. If one strategy dominates all others, it is called a **dominant strategy**.
- **Equilibrium**: no incentive for a player to deviate from their strategy.
- If someone has incentive to deviate, then the action profile does not form an equilibrium and therefore will not be played

A strategy profile made of dominant strategies for every player is a **Nash equilibrium**

Prisoner's Dilemma

- Proposed in 1950 by Melvin Dresher and Merrill Flood (RAND Corp), then given the prisoner's backstory by Albert Tucker (Princeton) during a seminar
- Classic example: two prisoners are given choice to confess or not confess (alternatively, cooperate vs. defect)

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- Classic example: two prisoners are given choice to confess or not confess (alternatively, cooperate vs. defect)
- Optimal solution is not "optimal"...so what do we mean by optimal?

Pareto Optimality

- From the point of view of an outside observer, are some outcomes better than others?
- An outcome is **non-Pareto optimal** if there exists another outcome which would give both players higher payoffs, or would give one player the same payoff, and the other, a higher payoff.
- Otherwise, the outcome is **Pareto optimal**.

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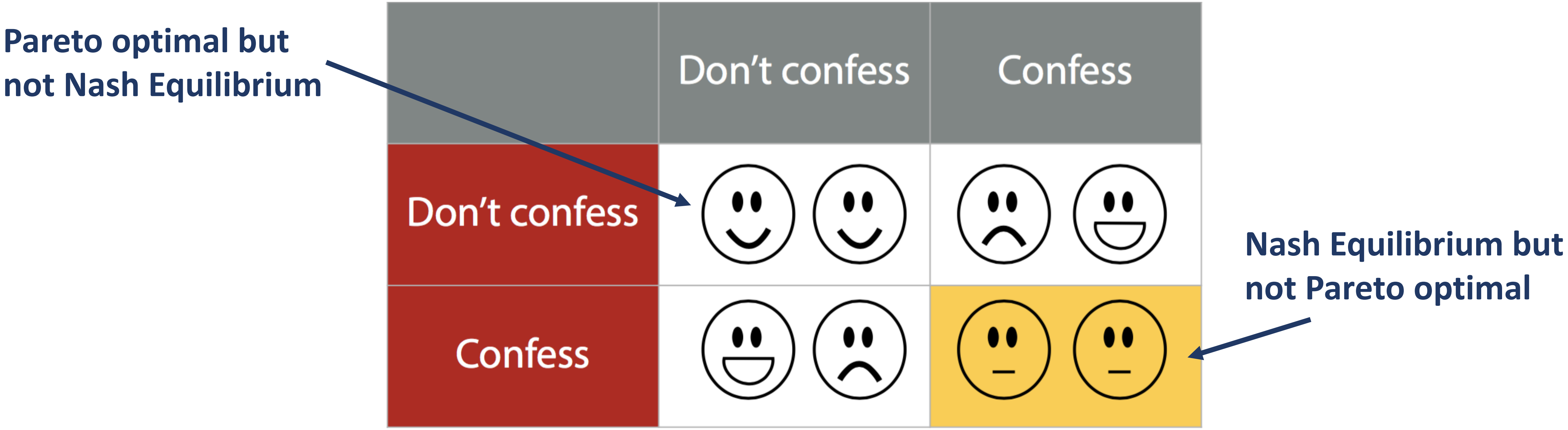

Pareto Optimality

Pareto Principle
To be acceptable as a solution, an outcome should be Pareto optimal