Problem Set 5: Game Theory

(Due Friday 3/17/2017 at 5pm)

Problem 1

(a) Consider the following partial-conflict game played in a non-cooperative manner. The first payoff is to player 1 (the row player – marked red); the second payoff is to player 2 (the column player).

	player 2	
player 1	Choice A	Choice B
Choice A	(6,1)	(3, 3)
Choice B	(5,5)	(1,6)

This game has a pure-strategy Nash equilibrium. What is it (explain why)? Is it Pareto optimal (explain)?

(b) If instead of the game matrix above, we have the following matrix:

	player 2		
player 1	Choice A	Choice B	
Choice A	(1,6)	(3, 3)	
Choice B	(5,5)	(6,1)	

What is the Nash equilibrium now (explain why)? Is it Pareto optimal (explain)?

Problem 2

In the following two-person zero-sum game, the payoffs shown in the matrix represent the gains of player 1 (row player) and the losses of player 2 (column player)

	player 2			
player 1	Choice A	Choice B	Choice C	Choice D
Choice A	(<mark>8</mark> ,-8)	(1, -1)	(5,-5)	(2,-2)
Choice B	(5,-5)	(4,-4)	(7,-7)	(<mark>8</mark> ,-8)
Choice C	(1 ,-1)	(3 ,-3)	(4 ,-4)	(5 ,-5)
Choice D	(<mark>8</mark> ,-8)	(4,-4)	(<mark>3</mark> ,-3)	(5 ,-5)

- (a) What is the **maximin** strategy of player 1 (maximin is the strategy in which player 1 maximizes their gains)?
- (b) What is the **minimax** strategy of player 2 (minimax is the strategy in which player 2 minimizes their losses)?

(c) Does the game have a saddle point? If it does, find it. If not, explain why not.

Problem 3

Consider the new hit television gameshow called Faith. You have two boxes, box 1 contains \$1,000, and box 2 contains either \$1,000,000 or nothing. You have observed the host to be 99.98% accurate in the last 10,000 games. If the host predicts that the contestant chooses only box 2, he rewards their faith with the million dollars. The following table summarizes all possible gains of the contestant:

	Host		
Contestant	Predicts that you select both boxes	Predicts that you select box 2	
You select both	\$1,000	\$1,001,000	
You select box 2	0	\$1,000,000	

Should you choose both boxes, or only box 2? To answer this question, answer the following:

- (a) What is the expected payoff if you take both boxes?
- (b) What is the expected payoff if you take box 2?
- (c) Based on the expected value principle, should you choose both boxes or only box 2?
- (d) Based on the dominance principle, should you choose both boxes or only box 2?
- (e) What would you do if you were to play this game?

Problem 4

Walker and Wooders (2001) Minimax Play at Wimbledon, The American Economic Review

The table below contains data from Wimbledon matches. The data represent probabilities of whether the server or the receiver win a point. If the server serves to the left and the receiver moves to the left, there is a 58% chance that the server wins the point, if the receiver moves to the right, the server has a 79% chance of winning the point, etc.

	receiver	
server	Left	Right
Left	0.58, 0.42	0.79, 0.21
Right	0.73 , 0.27	0.49 , 0.51

- (a) What is the optimal (mixed) strategy for the server?
- (b) What is the optimal (mixed) strategy for the receiver?