

Dualities of Stably Compact Spaces

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Stably compact spaces have attracted increased attention in recent years for a variety of reasons. First of all they appear to be the closest analog in the T_0 -setting to that fundamental topological class consisting of the compact Hausdorff spaces. Although they do not appear in the literature with the high frequency of compact Hausdorff spaces, yet they have a distinctive and substantial theory that is in many ways analogous to that of compact Hausdorff spaces and in some other ways is more interesting and intricate, for example, the theory of their associated partial orders. A second feature that has proved important from the perspective of theoretical computer science is the ability to transfer a variety of constructions that have arisen in domain theory to the setting of stably compact spaces and the resulting stability or robustness of the property of being stably compact under such constructions. Thus the “stably” of “stably compact” has broader connotations than its original definition. We note, however, that in moving from traditional domain theory to a theory centered on stably compact spaces, one often needs to replace order-theoretic structures with analogous topological ones, i.e., one is pushed from an order-theoretic to a topological perspective. However, strong connections remain between the theory of stably compact spaces and aspects of domain theory, which we will seek to emphasize.

A third feature of stably compact spaces is a basic duality which they themselves exhibit, the so-called de Groot duality, which manifests itself in various guises in the constructions that one carries out on them. There are various formulations of this basic duality and a variety of such extended dualities arising from it. For example, in the setting of stably compact spaces, certain dualities of an intuitive or informal nature such as angelic vs. demonic nondeterminism become explicit. More recent and exotic extended dualities involving capacities have been established by Jean Goubault-Larrecq. We suggest the measure-theoretic idea of input/output pavings as an alternative formulation of de Groot duality and a convenient framework for these more exotic dualities.