Categories of Formal Contexts

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Formal contexts (or *polarities* in Birkhof's terminology) provide a convenient combinatorial way to present closure operators on sets. They have been studied extensively for their applications to concept analysis, particularly for finite contexts, and are used regularly as a technical device in general lattice theory (for example, to describe the Dedekind-MacNeille completion of a lattice). In the most common uses, *morphisms* of contexts do not play a role. Although various scholars (most thoroughly, Marcel Erné) have considered certain notions of context morphisms, these efforts have generally concentrated either on special kinds of contexts that closely match certain "nice" lattices or on special kinds of lattice morphisms.

Here we propose a category of formal contexts in which morphisms are relations that satisfy a certain natural combinatorial property. The idea is to take our cue from the fact that a formal context is simply a binary relation between two sets. So the identity morphism of such an object should be that binary relation itself. From this, we get the combinatorial properties of morphisms more or less automatically.

The first main result of the talk is that the category of contexts is dually equivalent to the category INF, of complete meet lattices with meet-preserving maps. To get a duality with complete lattices, the second result characterizes those context morphisms that correspond to complete lattice homomorphisms.

We also consider various constructions that are well-known in INF to illustrate that formal contexts yield remarkably simple, combinatorial descriptions of many common constructions.