Sheaf Representations of MV-algebras via Priestley Duality

Mai Gehrke
Université Paris Diderot - Paris 7
Joint work with S. van Gool and V. Marra

MV-algebras, introduced by Chang in 1958 as semantics for the infinite-valued Lukasiewicz logic, have been studied extensively for their own sake and relative to their connections to various other areas of mathematics. Representation theory has played an important role in the theory of MV-algebras while duality theory has not been used except in some special subclasses such as locally finite or finitely presented MV-algebras. From the point of view of duality theory, the full class of MV-algebras provides an interesting case study because a simple split between topological and discrete components of duality is obstructed for a different reason than the one known from modal logic. This was the topic of several joint papers with Hilary Priestley in which we developed extended dualities, based on Priestley duality, for a type of varieties of distributive lattice ordered algebras including the variety of MV-algebras. However, in this talk I will mainly focus on more recent work joint with Sam van Gool and Vincenzo Marra. We study the lattice duals of arbitrary MV-algebras. Thus none of the special dualities developed by MV-algebraists apply. We show that the two representation theorems for arbitrary MV-algebras as algebras of global sections of sheaves of MV-algebras with special properties, obtained, respectively, by Filipoiu and Georgescu in 1995 and by Dubuc and Poveda in 2010, may be seen as stemming from decompositions of the lattice duals. Further, we use our analysis of the dual spaces of MV-algebras to establish a broad MV-algebraic generalisation of Kaplansky's classical 1947 result stating that a compact Hausdorff space is uniquely determined up to homeomorphism by its lattice of real-valued continuous functions. In this work, the theory of canonical extensions plays an important role in the analysis of the interaction between the lattice dual spaces and the multiplication of MV-algebras.