A Non-commutative Priestley Duality

Sam van Gool
Radboud University Nijmegen
Joint work with Andrej Bauer, Karin Cvetko-Vah, Mai Gehrke and Ganna Kudryavtseva.

In this talk on our paper [2], we describe a new duality for skew distributive lattices. *Skew lattices* [4] consist of two idempotent semigroup operations on the same underlying set, which are related by certain absorption identities, but are not necessarily commutative. Skew (distributive) lattices form a natural non-commutative generalization of (distributive) lattices. For more details on the algebraic properties of skew lattices, see, e.g., Karin Cvetko-Vah's lecture in this conference.

It is known [5] that so-called *left-handed skew distributive lattices* can be embedded in partial function algebras, whose operations are given by "*restriction*" and "*override*". We strengthen this result by proving that a category of left-handed skew distributive lattices with zero is in a dual equivalence with a category of *sheaves, or étalé spaces, over local Priestley spaces*. In particular, every skew distributive lattice is isomorphic to an *algebra of local sections* over the compact open downsets of a local Priestley space.

Although our duality for left-handed skew distributive lattices is not a natural duality in the sense of Clark, Davey, Priestley et al., it does make use of the idea that the *points* of the dual space of a given algebra should correspond to certain *quotients* of that algebra. In the case of left-handed skew distributive lattices, a point of the dual space can be regarded as an element of quotient of the left-handed skew distributive lattice which is 'primitive'. Here, the primitive left-handed skew distributive lattices form an easy to understand, but infinite, subclass of left-handed skew distributive lattices, which generates the entire variety.

Our results generalize both Priestley duality [6] and the recent development of Stone duality for skew Boolean algebras [1, 3].

References

[1] A. Bauer and K. Cvetko-Vah, Stone duality for skew Boolean algebras with intersections, to appear in Houston J. Math. (2011), (arxiv.org:1106.0425).

[2] A. Bauer, K. Cvetko-Vah, M. Gehrke, S. van Gool, and G. Kudryavtseva, A non-commutative Priestley duality, submitted to Topology and its Applications (2012), (arxiv:1206.5848).

[3] G. Kudryavtseva, A refinement of Stone duality to skew Boolean algebras, to appear in Alg. Universalis (2011), (arxiv:1102.1242).

[4] J. Leech, Skew lattices in rings, Alg. Universalis 26 (1989), 48-72.

[5] J. Leech, Normal Skew Lattices, Semigroup Forum 44 (1992), 1-8.

[6] H. Priestley, Representation of distributive lattices by means of ordered Stone spaces, Bulletin of the London Mathematical Society 2 (1970), 186-190.