Mundici's Γ-functor Theorem for Star-shaped Sets via Minkowski's Duality with Gauge Functions

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Extending previous results by C. C. Chang, in 1986 D. Mundici proved that to any MV-algebra A there corresponds a uniquely determined lattice-ordered Abelian group with unit (G, u) such that A is (isomorphic to) the unit interval $[0, u]$ of (G, u). The functor that takes (G, u) back to $[0, u]$ is traditionally denoted Γ. The construction of (G, u) from A uses "good sequences" of elements in A that suffice to canonically represent each element of G. We consider here the special case that G is the vector lattice (that is lattice-ordered vector space) of all continuous real-valued positively homogeneous functions on $\mathbb{R}^n$, with unit a distinguished such function that is nowhere zero except at the origin. We first generalise the classical theory of Minkowski's gauge functions of convex bodies so as to obtain an order-reversing bijection between the set of appropriately defined "star-shaped bodies" in $\mathbb{R}^n$, and the elements in the positive cone of G. We are then able to prove geometrically a representation theorem for star-shaped bodies that, through the order-reversing bijection above, is seen to be the exact geometric counterpart of the Γ-functor theorem. These results are part of the speaker's research towards her Ph.D. diploma. The investigation is concerned with the construction of algebraic counterparts, in the braad sense of duality theory, of convex geometry in Euclidean spaces.