The Many-valued Coalgebraic Cover Modality

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The aim of Coalgebraic Logic is to find formalisms that allow reasoning about $T$-coalgebras uniformly in the functor $T$. Moss’ seminal idea was to consider the set functor $T$ as providing a modality $\nabla_T$, the semantics of which is given in terms of the relation lifting of $T$. The latter exists whenever $T$ preserves weak pullbacks.

In joint work with Marta Bílková, Alexander Kurz and Jiří Velebil, we introduce basic notions and results about relation liftings on categories enriched in a commutative quantale. We derive two necessary and sufficient conditions for a 2-functor $T$ to admit a functorial relation lifting: one is the existence of a distributive law of $T$ over the ‘powerset monad’ on categories, one is the preservation by $T$ of ‘exactness’ of certain squares. Both characterizations are generalizations of classical results known for set functors: the first characterization generalizes the existence of a distributive law over the genuine powerset monad, the second generalizes preservation of weak pullbacks.

We then introduce a generalized power set functor on categories enriched in commutative quantales. Under certain assumption on the quantale, this functor admits a relation lifting and allows us to study the semantics of the coalgebraic cover modality in the enriched setting.