

Topological duality for lattices via canonical extensions

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Joint work with Mai Gehrke

In this talk on our forthcoming paper [4], we describe a new topological duality for bounded lattices. The two main features of our duality are that it generalizes Stone duality for bounded distributive lattices, and that the morphisms on either side are not the standard ones. A positive consequence of the choice of morphisms is that those on the topological side are functional. We obtain the following results:

- (a) canonical extensions of bounded lattices are the algebraic versions of the existing dualities for bounded lattices by Urquhart [7] and Hartung [5];
- (b) there is a universal construction which associates to an arbitrary lattice two distributive lattice envelopes with an adjoint pair between them;
- (c) we identify precisely which maps between bounded lattices admit functional duals on our newly defined dual spaces.

For the result mentioned under (a), we rely on previous work of Gehrke, Jónsson and Harding [2,3]. For the universal construction of (b), we modify a construction of the injective hull of a semilattice by Bruns and Lakser [1], adjusting their concept of ‘admissibility’ to the finitary case. For (c), we use Priestley duality for distributive lattices [6] and our own universal characterization in (b) of the distributive envelopes a bounded lattice.

[1] G. Bruns and H. Lakser, *Injective hulls of semilattices*, Canadian Mathematical Bulletin **13** (1970), no. 1, 115-118.

[2] Mai Gehrke and John Harding, *Bounded lattice expansions*, Journal of Algebra **238** (2001), no. 1, 345-371.

[3] Mai Gehrke and Bjarni Jónsson, *Bounded distributive lattices with operators*, Mathematica Japonica **40** (1994), no. 2, 207-215.

[4] Mai Gehrke and Sam van Gool, *Topological duality for lattices via canonical extensions*, forthcoming (2012).

[5] Gerd Hartung, An extended duality for lattices, *General algebra and applications* (K. Deicke and H.J. Vogel, eds.), Heldermann-Verlag, Berlin, 1993, pp. 126-142.

[6] Hilary Priestley, *Representation of distributive lattices by means of ordered Stone spaces*, Bulletin of the London Mathematical Society **2** (1970), 186-190.

[7] Alasdair Urquhart, *A topological representation theory for lattices*, Algebra Universalis **8** (1978), no. 1, 45-58.