A new model for polythermal ice
incorporating gravity-driven meltwater drainage

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I. What do models tell us about how subglacial discharge is delivered at grounding lines?

II. How does the spatial distribution of subglacial discharge affect the shape of ice shelves?

Subglacial channels have a ‘trumpet-like’ shape near the margin
A new model for polythermal ice
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Motivation

**Goal** - provide a simple model that

- predicts temperature and water content of polythermal ice.
- allows water to drain from the ice by porous flow.

- Ice flow depends on temperature and water content.
- May be fast dynamical feedbacks between water content and ice flow.
Motivation
Previous work on polythermal ice

**Theory**
- Lliboutry (1971, 1976), Nye & Frank (1973) - permeability
- Fowler & Larson (1978) - continuum formulation, no moisture movement
- Hutter (1982) - mixture theory, diffusive moisture transport
- Fowler (1984) - two-phase theory, Darcy’s law for moisture transport

**Computational models**
- Greve (1997) - two layer, explicit determination of ‘CTS’, switch-like drainage function
Problem formulation

**Stokes flow** (or approximation)

\[
\nabla \cdot \mathbf{u} = 0 \\
\frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial p}{\partial x_i} = -\rho g_i
\]

\[
\tau_{ij} = A^{-1/n} \varepsilon^{1/n-1} \dot{\varepsilon}_{ij}
\]

\[
A = A(T, \phi) \quad \phi = \text{water content (porosity)}
\]

**This talk** - ice velocity prescribed (decoupled from thermodynamics)
Problem formulation

**Energy conservation**

**Cold ice**

\[
\rho c \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \tau_{ij} \dot{e}_{ij}, \quad \phi = 0, \quad T \leq T_m
\]

**Temperate ice**

\[
\rho_w L \left( \frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi \right) + \rho_w L \nabla \cdot \mathbf{j} = \tau_{ij} \dot{e}_{ij}, \quad T = T_m, \quad \phi > 0
\]

Relative water flux  \( \mathbf{j} = -\nu \nabla \phi \) - enthalpy gradient method

Schoof & Hewitt 2015 *in review*
Problem formulation

Energy conservation

In terms of ‘enthalpy’,
\[ h = \rho cT + \rho_w L_\phi \]

\[ \left( \frac{\partial h}{\partial t} + \mathbf{u} \cdot \nabla h \right) + \nabla \cdot \mathbf{Q} = \tau_{ij} \dot{\varepsilon}_{ij}, \quad \mathbf{Q} = \begin{cases} -k \nabla T & h < \rho c T_m, \\ \rho_w L j & h \geq \rho c T_m. \end{cases} \]
Problem formulation

Energy conservation

In terms of ‘enthalpy’,
\[ h = \rho c T + \rho_w L \phi \]

Relative moisture flux (Darcy’s law)
\[ j = \frac{k_0 \phi^2}{\eta_w} (\rho_w g - \nabla p_w) \]

\[ \nabla p \approx \rho g \]

\[ \nabla \cdot \mathbf{Q} = \tau_{ij} \dot{e}_{ij}, \quad \mathbf{Q} = \begin{cases} -k \nabla T & h < \rho c T_m, \\ \rho_w L j & h \geq \rho c T_m. \end{cases} \]
Problem formulation

Energy conservation

In terms of ‘enthalpy’,
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\[ j = \frac{k_0 \phi^2}{\eta_w} (\rho_w g - \nabla p_w) \]

Viscous compaction  
e.g. Hewitt & Fowler 2008
\[ \nabla \cdot j = \frac{\phi p_e}{\eta} \]
\[ j = \frac{k_0 \phi^\alpha}{\eta_w} ((\rho_w - \rho) g + \nabla p_e) \]

Permeability (?)

Effective pressure
\[ \nabla p \approx \rho g \]

Pore pressure  
\[ p_w = p - p_e \]
Ice divide example

- Pore pressure = subglacial drainage pressure \( p_e = N_0 \)

- Velocity from shallow ice approximation (thermodynamically decoupled).
Ice divide example

‘Standard’ enthalpy gradient method

Compaction pressure method
Slab glacier test case

Slab glacier test case

- Comparison of different models

- Standard enthalpy gradient well approximates no-water-transport solution.
- Gravity-driven drainage results in less temperate ice - and some interesting behaviour…
Ice divide example

Cold-temperate transition

Meltwater flux to bed

‘Standard’ enthalpy gradient model

Compaction pressure model
Summary

- Suggested a simple model to incorporate polythermal ice in existing ice-sheet models - alternative to enthalpy gradient method.

- Model allows water transport through the ice, and connection with subglacial drainage - but more knowledge of permeability needed.

- Worth exploring dynamics of temperate ice + subglacial water + sediments further.