Geometry Advanced Class HT25

Organizers: Lukas Brantner and Dominic Joyce

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Subject: Pantev-Toen-Vaquie-Vezzosi shifted symplectic derived algebraic geometry, and subsequent developments.

Time and place: we will hold meetings Fridays 9.30-11.0 in weeks 1 and 3-8 in HT 25, in rooms: week 1 (24/1/25) C4, week 3 (7/2/25) C5, week 4 (14/2/25) C4, week 5 (21/2/25) C5, week 6 (28/2/25) C4, week 7 (7/3/25) C5, week 8 (13/3/25) C1.

Introduction
In 2011, in the context of Toen-Vezzosi's theory of Derived Algebraic Geometry, Pantev-Toen-Vaquie-Vezzosi https://arxiv.org/abs/1111.3209 introduced the notion of 'k-shifted symplectic structure' on a derived scheme or derived stack. They showed that derived moduli stacks of coherent sheaves on a Calabi-Yau m-fold have $(2-m)$-shifted symplectic structures. They also defined 'k-shifted Lagrangians' in k-shifted symplectic stacks.

  The theory has been applied in Donaldson-Thomas theory and its generalizations, which study enumerative invariants of coherent sheaves on Calabi-Yau 3- or 4-folds. It has been developed in several important ways since the original paper.

  This class will build on the reading group run by Chenjing Bu and Andres Ibanez Nunez in TT24, though you do not need to have gone to this. As the subject is very technical and tends to long papers, we will not aim to present results in detail, but instead to provide a survey or broad overview. The idea is that each week, a volunteer will explain an important paper in the field.

Practical details
Meetings will be weekly in HT25, excluding week 2, on Fridays at 9.30am, and will last 90 minutes. For the first meeting, Dominic will give a brief introduction to derived stacks and PTVV shifted symplectic structures, and an overview of their application in generalizations of Donaldson-Thomas theory. We will then draw up a timetable for the rest of term, asking for volunteers to talk on some subject each week. If there is interest in this, we may make the last meeting an open problem session, in which anyone can propose interesting open problems for discussion.

  Speakers are asked to prepare 80 minute talks. They are also encouraged to provide notes (could be handwritten, or PDF slides) which will be posted on a class webpage, but we don't insist on this if speakers feel it would be too much work. Alternatively, a designated note-taker in the audience could provide notes afterwards.

Provisional programme:
Week 1: Dominic, introduction.
Week 3: Chenjing, on Calaque-Haugseng-Scheimbauer, 'The AKSZ Construction in Derived Algebraic Geometry as an Extended Topological Field Theory'.
Week 4: Another volunteer (preferable), or Dominic on CY4 virtual classes.
Week 5: Nick Kuhn.

Week 6: ? Sam Moore, on Blanc-Katzarkov-Pandit, 'Generators in formal deformations of categories'.
Week 7: Lukas, on something exciting.
Week 8: Open problem session.

Papers we could cover during the class, and useful sources

Pantev-Toen-Vaquie-Vezzosi, 'Shifted symplectic structures', https://arxiv.org/abs/1111.3209.

Calaque-Pantev-Toen-Vaquie-Vezzosi, 'Shifted Poisson structures and deformation quantization', https://arxiv.org/abs/1506.03699.

Calaque-Haugseng-Scheimbauer, 'The AKSZ Construction in Derived Algebraic Geometry as an Extended Topological Field Theory', https://arxiv.org/abs/2108.02473.

Haugseng-Melani-Safronov, 'Shifted coisotropic correspondences', https://arxiv.org/abs/1904.11312.

Brav-Dyckerhoff, 'Relative Calabi-Yau structures, I, II', https://arxiv.org/abs/1606.00619, https://arxiv.org/abs/1812.11913.

Calaque, Safronov, 'Shifted cotangent bundles, symplectic groupoids, and deformation to the normal cone', https://arxiv.org/abs/2407.08622.

Katzarkov-Pandit-Spaide, 'Calabi-Yau structures, spherical functors, and shifted symplectic structures', https://arxiv.org/abs/1701.07789.

Blanc-Katzarkov-Pandit, 'Generators in formal deformations of categories', https://arxiv.org/abs/1705.00655.

A. Khan, 'An introduction to derived algebraic geometry', https://www.preschema.com/lecture-notes/2023-kias/dagkias.pdf.

H. Park, 'Shifted symplectic pushforwards', https://arxiv.org/abs/2406.19192.

T. Pantev and B. Toën, 'Moduli of flat connections on smooth varieties', https://arxiv.org/abs/1905.12124.