Section c Algebraic Topology

Hilary 2003

Question Sheet 5

1. Prove directly that the homology groups of $\mathbb{R}^n$ are the same as those of a point.

2. Find singular cycles in the following spaces. Hence formulate conjectures about their homology groups:

   (i) $S^1 \vee S^1$

   (ii) $\mathbb{R}^2 - \{0\}$

   (iii) the Klein bottle

   (iv) the universal cover of $S^1 \vee S^2$.

3. If $f: X \to Y$ is a map and $f(x_0) = y_0$ then show that the diagram

$$\begin{array}{ccc} \pi_1(X, x_0) & \xrightarrow{f_*} & \pi_1(Y, y_0) \\ \downarrow h & & \downarrow h \\ H_1(X) & \xrightarrow{f_*} & H_1(Y) \end{array}$$

   commutes.

4. If $f: \tilde{X} \to X$ is a covering map, is it true that $f_*: H_1(\tilde{X}) \to H_1(X)$ is injective? Give either a proof or counterexample.