Rational points on curves:
a p-adic and computational perspective

Monday

Kartik Prasanna (9:30 - 10:30)

p-adic L-functions and the coniveau filtration on Chow groups

Let E be the elliptic curve $X_0(49)$, which has complex multiplication by the ring of integers of the imaginary quadratic field with discriminant -7. I will explain a proof of part of the Bloch-Beilinson conjecture for arbitrary powers of the Hecke character associated to E; namely, whenever a power of this Hecke character gives rise to a self-dual motive with vanishing central L-value, we construct a variety V over $\mathbf{Q}$ realizing this motive and a corresponding element in the Chow group of homologically trivial cycles on V that is provably nontorsion. In fact, we show also that this cycle is nonzero in the top graded piece for the coniveau filtration on the Chow group (with $\mathbf{Q}$-coefficients), and in particular no multiple of it is algebraically equivalent to zero. (Joint work with Bertolini/Darmon.)

Jennifer Balakrishnan (11:00 - 11:40)

Applications of explicit Coleman integration on hyperelliptic curves

We discuss methods to compute iterated Coleman integrals on hyperelliptic curves. In particular, we present some applications to the computation of rational points on these curves via Kim's nonabelian analogue of the Chabauty method.

Robert Pollack (11:45 - 12:25)

Families of overconvergent modular symbols

Overconvergent modular symbols form an infinite-dimensional Banach space which contains all systems of Hecke-eigenvalues of overconvergent modular forms as well as the p-adic L-functions attached to these forms. In a previous joint work with Glenn Stevens, we gave an efficient algorithm which computes the overconvergent modular symbol attached to a classical modular form, and in particular, gave an efficient algorithm for computing p-adic L-functions. (These algorithms can also be used to compute Stark-Heegner points on elliptic curves.)

In this talk, we seek to generalize these algorithms to families of overconvergent modular symbols. Such a generalization would, for example, allow us to compute families of Hecke-eigenvalues,two-variable p-adic L-functions, as well as the Lambda-structure of Hida (Hecke) algebras.

Xevi Guitart (2:20 - 3:00)

Numerical computation of Stark-Heegner points in higher level

Let E be an elliptic curve over $\mathbf{Q}$ of conductor pM, and let K be a real quadratic field in which the prime p is inert and all primes dividing M are split. In this context, Stark-Heegner points on $E(K_p)$ can be computed as the image under Tate's uniformization map of certain p-adic periods, which are defined in terms of semidefinite integrals. The direct calculation of semidefinite integrals appears to be difficult, but if E has conductor p they can beexpressed in terms of definite integrals, which can be then efficiently computed using overconvergent modular symbols. The algorithm for going from semidefinite to definite integrals exploits the continued fraction expansion of rational cusps.In this talk I will discuss a variation of this "continued fraction trick", which allows for the expression of semidefinite integrals in terms of definite integrals also in curves of non-prime conductor, so that Stark-Heegner points can also be numerically computed using overconvergent modular symbols. In fact, a similar method also applies to ATR points, which are a different (non p-adic) type of Stark-Heegner points on curves over real quadratic fields. In that framework, the algorithm can be used for computing on curves of non-trivial conductor. This is joint work with Marc Masdeu.

John Voight (3:05 - 3:45)

Semi-arithmetic points

We present a method for constructing algebraic points on elliptic curves defined over number fields, combining the theory of Belyi maps and quaternionic Shimura varieties; our method generalizes the construction of Heegner points arising from classical modular curves. In particular, we report on some computational investigations of these points.

Shinichi Kobayashi (4:20 - 5:00)

The p-adic Gross-Zagier formula at supersingular primes

The p-adic Gross-Zagier formula relates the derivative of the p-adic L-function with the p-adic height of the Heegner point. It also has an application for the full Birch and Swinnerton-Dyer conjecture. I explain the formula and its proof especially at supersingular primes.

Tuesday

Toby Gee (9:30 - 10:30)

Patching functors and the cohomology of Shimura curves

I will explain recent joint work with Matthew Emerton and David Savitt, in which we relate the geometry of various tamely potentially Barsotti–Tate deformation rings for two-dimensional Galois representations to the integral structure of the cohomology of Shimura curves. As a consequence, we establish some conjectures of Breuil regarding this integral structure.

The key technique is the Taylor–Wiles–Kisin patching argument, which, when combined with a new, geometric perspective on the Breuil–Mezard conjecture, forges a tight link between the structure of cohomology (a global automorphic invariant) and local deformation rings (local Galois-theoretic invariants).

Shin Hattori (11:00 - 12:00)

Canonical subgroups via Breuil-Kisin modules

The overconvergence of the canonical subgroup of the universal abelian variety is one of the key ingredients of the theory of overconvergent modular forms. In this talk, I will show the overconvergence of the canonical subgroup with expected properties via the Breuil–Kisin classification, including the case of $p = 2$.

Yuichiro Taguchi (2:00 - 2:40)

Rational torsion points of abelian varieties over a large extension of a local field

We extend the following theorem of H. Imai in several ways: If A is an abelian variety with potentially good reduction over a finite extension K of $\mathbf{Q}_p$, then it has only finitely many rational torsion points over the maximal p-cyclotomic extension of K. In particular, we prove the finiteness over $K(K^{1/p^\infty})$.

Steffen Müller (2:45 - 3:25)

A p-adic BSD conjecture for modular abelian varieties

In 1986 Mazur, Tate and Teitelbaum came up with a p-adic analogue of the conjecture of Birch and Swinnerton-Dyer for elliptic curves over the rationals. In this talk I will report on joint work with Jennifer Balakrishnan and William Stein on a generalization of this conjecture to the case of modular abelian varieties and primes p of good ordinary reduction. I will discuss the theoretical background that led us to the formulation of the conjecture, as well as numerical evidence supporting it in the case of modular abelian surfaces and the algorithms that we used to gather this evidence.

Valentina di Proietto (4:00 - 4:40)

On the p-adic invariant cycles theorem

For a proper semistable curve over a DVR of mixed characteristics we reprove the "invariant cycles theorem" with trivial coefficients by Chiarellotto i.e. that the group of elements annihilated by the monodromy operator on the first de Rham cohomology group of the generic fiber coincides with the first rigid cohomology group of the special fiber, without the hypothesis that the residue field is finite. This is done using the explicit description of the monodromy operator on the de Rham cohomology of the generic fiber with coefficients convergent F-isocrystals given in a work of Coleman and Iovita. We apply these ideas to the case where the coefficients are unipotent convergent F-isocrystals defined on the special fiber: we show that the invariant cycles theorem does not hold in general in this setting. Moreover we give a sufficient condition for the non exactness. It is a joint work with B. Chiarellotto, R. Coleman and A. Iovita.

Francesco Baldassarri (4:45 - 5:25)

Radius of convergence of p-adic connections and the Berkovich ramification locus

We apply the theory of the radius of convergence of a p-adic connection to the special case of the direct image of the constant connection via a finite morphism of compact p-adic curves, smooth in the sense of rigid geometry. We show that a trivial lower bound for that radius implies a global form of Robert's p-adic Rolle theorem. The proof is based on a widely believed, although unpublished, result of simultaneous semistable reduction for finite morphisms of smooth p-adic curves. We also clarify the relation between the notion of radius of convergence used in our previous work and the more intrinsic one used by Kedlaya. (The paper is available at http://arxiv.org/abs/1209.0081)

Wednesday

Glenn Stevens (9:15 - 10:15)

The Hodge-Tate sequence and overconvergent p-adic modular sheaves

Using Faltings' theory of the Hodge-Tate sequence of an abelian scheme we construct certain sheaves Ω^κ, where κ is a not-necessarily integral weight, over formal subschemes of modular varieties over which the canonical subgroup exists. These sheaves generalize the integral powers, ω^k, of the sheaf ω of relative differentials on a modular curve. Global sections of Ω^κ provide geometric realizations of overconvergent automorphic forms of non-integral weight. Applications of this approach to the theory of p-adic Hilbert modular forms will be given. This is joint work with Fabrizio Andreotti and Adrian Iovita.

Adrian Iovita (10:45 - 11:45)

An overconvergent Eichler-Shimura isomorphism

Given a p-adic weight and a finite slope we describe a Hecke and Galois equivariant geometric map relating elliptic overconvergent modular symbols and overconvergent modular forms of that slope, appropriate weights and $\mathbf{C}_p$-coefficients. We show that for a fixed slope, with the possible exception of a discrete family of weights, this map is an isomorphism.

Matt Greenberg (11:50 - 12:30)

Triple product p-adic L-functions for balanced weights

In this talk, I will describe a construction of a p-adic L-function attached to a triple of p-adic Coleman families of cusp forms. This function interpolates algebraic parts of special values of Garrett triple product L-functions at balanced triples of weights. Our construction is complementary to that of Harris and Tilouine which treats the case of unbalanced weights.

Thursday

Anna Cadoret (9:30 - 10:30)

ℓ-adic representations of etale fundamental group of curves

I will present an overview of a series of joint works with Akio Tamagawa about l-adic representations of etale fundamental group of curves (to simplify, over finitely generated fields of characteristic 0).

More precisely, when the generic representation is GLP (geometrically Lie perfect) i.e. the Lie algebra of the geometric etale fundamental group is perfect, we show that the associated local ℓ-adic Galois representations satisfies a strong uniform open image theorem (ouside a 'small' exceptional locus). Representations on l-adic cohohomology provide an important example of GLP representations. In that case, one can even prove that the exceptional loci that appear in the statement of our strong uniform open image theorem are independent of ℓ, which was predicted by motivic conjectures.

Without the GLP assumption, we prove that the associated local l-adic Galois representations still satisfy remarkable rigidity properties: the codimension of the image at the special fibre in the image at the generic fibre is at most 2 (outside a 'small' exceptional locus) and its Lie algebra is controlled by the first terms of the derived series of the Lie algebra of the image at the generic fibre.

I will state the results precisely, mention a few applications/open questions and draw a general picture of the proof in the GLP case (which, in particular, intertwins via the formalism of Galois categories, arithmetico-geometric properties of curves and ℓ-adic geometry). If time allows, I will also give a few hints about the ℓ-independency of the exceptional loci or the non GLP case.

Gunther Cornelissen (11:00 - 12:00)

Recovering curves from L-series

The main result of the talk is that two curves over a finite field are isomorphic, up to automorphisms of the ground field, if and only if there is an isomorphism of groups of Dirichlet characters such that the corresponding L-series are all equal. This can be shown by combining Uchida's proof of the anabelian theorem for global function fields with methods from (noncommutative) dynamical systems. I will also discuss how to turn this theorem into a theoretical algorithm that, given a listing of L-functions, determines an equation for the corresponding curve(s).

Victor Rotger (2:00 - 2:40)

Triple product p-adic L-functions and diagonal cycles

In this lecture I shall introduce certain generalised Gross-Kudla-Schoen diagonal cycles in the product of three Kuga-Sato varieties and a p-adic formula of Gross-Zagier type which relates the images of these diagonal cycles under the p-adic Abel-Jacobi map to special values of the p-adic L-function attached to

the Garrett triple convolution of three Hida families of modular forms. This formula has applications to the Birch–Swinnerton-Dyer conjecture and the theory of Stark-Heegner points. (Joint work with Henri Darmon.)

Massimo Bertolini (2:45 - 3:25)

p-adic Beilinson's formulas for Rankin p-adic L-functions and applications

I will report on p-adic Beilinson's formulas, relating the values of certain Rankin p-adic L-functions outside their range of classical interpolation, to p-adic syntomic regulators of Beilinson-Kato and Beilinson-Flach elements. Applications to the theory of Euler systems and to the Birch and Swinnerton-Dyer conjecture will also be discussed. This is joint work with Henri Darmon and Victor Rotger.

Samit Dasgupta (4:00 - 5:00)

Factorization of p-adic Rankin L-series

We show that the p-adic L-function associated to the tensor square of a p-ordinary eigenform factors as the product of the symmetric square p-adic L-function of the form with a Kubota-Leopoldt p-adic L-function. Our method of proof follows that of Gross, who proved a factorization for Katz's p-adic L-function for a character arising as the restriction of a Dirichlet character. We prove certain special value formulae for classical and p-adic Rankin L-series at non-critical points. The formula of Bertolini, Darmon, and Rotger in the p-adic setting is a key element of our proof. As demonstrated by Citro, we obtain as a corollary of our main result a proof of the exceptional zero conjecture of Greenberg for the symmetric square.

Friday

David Loeffler (9:30 - 10:30)

Euler systems for Rankin–Selberg convolutions of modular forms

I will describe a construction of special cohomology classes over the cyclotomic tower for the product of the Galois representations attached to two modular forms, which p-adically interpolate the "Beilinson–Flach elements" of Bertolini, Darmon and Rotger. I will also describe some applications to the Iwasawa theory of modular forms over imaginary quadratic fields.

Pierre Charollois (11:00 - 12:00)

Eisenstein cocycle on GL_n *and computation of p-adic L-functions of totally real fields*

We define an integral version of Sczech cocycle on $\mathrm{GL}_n(\mathbf{Z})$ by raising the level at a prime ℓ. As a result, we obtain a new construction of the p-adic L-functions of Barsky/Cassou-Noguès/Deligne-Ribet. This cohomological construction further allows for a study of the leading term of these L-functions at $s = 0$:
1) we obtain a new proof that the order of vanishing is at least the one conjectured by Gross. This was already known as result of Wiles.
2) we deduce an analog of the modular symbol algorithm for GL_n from the cocycle relation and LLL. It enables for the efficient computation of the special values of these p-adic L-functions.
When combined with a refinement of the Gross-Stark conjecture, we obtain some examples of numerical construction of $\mathfrak{p}$-units in class fields of totally real (cubic) fields. This is joint work with Samit Dasgupta.

Henri Darmon (2:00 - 3:00)

p-adic iterated integrals and rational points on elliptic curves

The p-adic Gross-Zagier formula for diagonal cycles and the p-adic Beilinson formulae described in the lectures of Rotger and Bertolini respectively suggest a connection between certain *p-adic iterated integrals* attached to modular forms and rational point on elliptic curves. I will describe an ongoing project (in collaboration with Alan Lauder and Victor Rotger) whose goal is to explore these relationships numerically, with the goal of better understanding the notion of *Stark-Heegner points*. It is hoped that these experiments might suggest new perspectives on Stark-Heegner points based on suitable *p-adic deformations* of the global objects–diagonal cycles, Beilinson-Kato and Beilinson-Flach elements–described in the lectures of Rotger, Bertolini, Dasgupta, and Loeffler, following the influential approach to p-adic L-functions pioneered by Coates-Wiles, Kato, and Perrin-Riou.

Alan Lauder (3:05 - 3:45)

Efficient computation of Rankin p-adic L-functions

I will present an efficient algorithm for computing certain special values of Rankin triple product p-adic L-functions and give an application of this to the explicit construction of rational points on elliptic curves.