Bernard Le Stum 19/03/10 9:30

Towards an overconvergent Deligne-Kashiwara correspondence

1) $/\mathbb{Q}_q$ $\quad U$ defined by $\{ y^2 = Q(x),\ \rho < |y| < \eta,\ |Q(x)| \cancel{<} > \eta \}$

Want $H^1_{dR}(U)$

Consider
$$\begin{array}{cc} (x,y) & U \\ \downarrow & \downarrow{\scriptstyle p} \\ x & V \end{array} \qquad x < \sqrt{\eta},\ |Q(x)| > \eta$$

$$p_* \mathcal{O}_U = \mathcal{O}_V \oplus \mathcal{K}$$

$$\Rightarrow H^1_{dR}(U) = H^1_{dR}(V) \oplus H^1_{dR}(V, \mathcal{K})$$

$$\mathcal{K} = \mathcal{O}_V \theta \qquad \qquad \Rightarrow \mathcal{K} \in MIC(V)$$
$$\nabla \theta = \theta \frac{dQ}{2Q} \qquad [\text{in fact, } \mathcal{K} \in M^\dagger(\overline{V})\,]$$

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2) $/\mathbb{C}$ $\quad X$ analytic manifold

abelian comparison: $\quad H^i_{dR}(X) \simeq H^i_{sing}(X, \mathbb{C})$

non-abelian comparison: $\quad MIC(X) \simeq LOC(X) \simeq Rep_{\mathbb{C}} \pi_1(X(\mathbb{C}), *)$

But these categories are not stable under, e.g., f_* if $f: Y \to X$.

$$\begin{array}{ccc} MIC(X) & & LOC(X) \\ \cap & & \cap \\ (\mathcal{D}_X\text{-mod})_{\substack{hol \\ reg}} & & (\mathbb{C}_X\text{-mod})_{cons} \\ \cap & & \cap \\ D^b_{\substack{hol \\ reg}}(\mathcal{D}_X) & \xrightarrow{\sim} & D^b_{cons}(\mathbb{C}_X) \leftarrow \text{perverse sheaves} \end{array}$$

1) /0