Ambitwistor Strings at Null Infinity and Asymptotic Symmetries

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Based on 1406.1462 (Geyer/Lipstein/Mason)

Overview

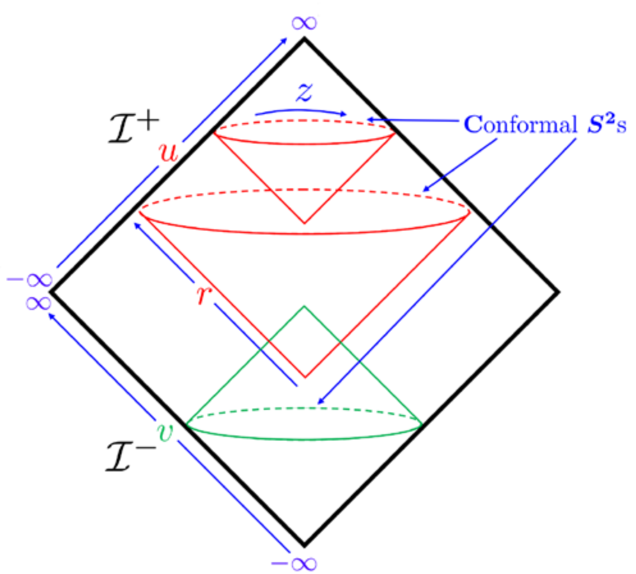
- Recently, Strominger and collaborators proposed a new way of understanding soft limit theorems in terms of asymptotic symmetries discovered by Bondi, Metzner, Sachs (BMS).

- Using ambitwistor string theory, which is a chiral infinite tension limit of the RNS string, these soft theorems can be proven from the perspective of conformal field theory and extended in various ways.

BMS Symmetry

- Strominger's conjecture: diag(BMS$^+$ x BMS$^-$) is a symmetry of the 4d gravitational S-matrix:

$$\langle out| B^+\mathcal{S} - \mathcal{S}B^- |in\rangle = 0$$



Soft Limits

- The Ward identities associated with BMS symmetry correspond to soft graviton theorems:

$$\lim_{k_{n+1}\to 0} \mathcal{M}_{n+1} = \left(S^{(0)} + S^{(1)}\right) \mathcal{M}_n$$

where

$$S^{(0)} = \sum_{a=1}^{n} \frac{(\epsilon \cdot k_a)^2}{s \cdot k_a} \longleftarrow \text{supertranslations}$$

$$S^{(1)} = \frac{\epsilon_{\mu\nu} k_a^{\mu} s_{\lambda} J_a^{\lambda\nu}}{s \cdot k_a} \longleftarrow \text{superrotations}$$

(Weinberg, White, Cachazo/Strominger)

Soft Gravitons

- A key step in Strominger's argument is that acting with a BMS generator on a state at null infinity leads to the insertion of a soft graviton. For concreteness, focus on supertranslations:

$$T^{-}\left|in\right\rangle = F^{-}\left|in\right\rangle + \sum_{k\in in} E_k f\left(z_k, \bar{z}_k\right)\left|in\right\rangle$$

$$\left\langle out\right| T^{+} = \left\langle out\right| F^{+} + \sum_{j\in out} E_j f\left(z_j, \bar{z}_j\right)\left\langle out\right|$$

- Plugging this into the Ward identity then gives

$$\left\langle out\right| F^{+}\mathcal{S} - \mathcal{S}F^{-}\left|in\right\rangle = \left(\sum_{k\in in} E_k f\left(z_k, \bar{z}_k\right) - \sum_{j\in out} E_j f\left(z_j, \bar{z}_j\right)\right)\left\langle out\right|\mathcal{S}\left|in\right\rangle$$

Generalizations

- Yang-Mills soft limits:

$$\mathcal{A}_{n+1} = \left(S^{(0)} + S^{(1)}\right) \mathcal{A}_n$$

$$S^{(0)} = \sum_{a \text{ adj. } s} \frac{\epsilon \cdot k_a}{s \cdot k_a}, \qquad S^{(1)} = \sum_{a \text{ adj. } s} \frac{\epsilon_\mu s_\nu J_a^{\mu\nu}}{s \cdot k_a}$$

Low, Burnett/Kroll, Casali

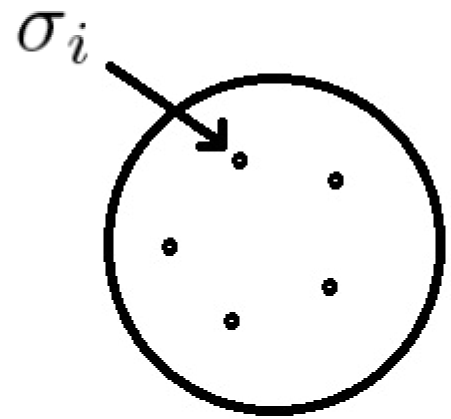
- Schwab/Volovich generalized the soft photon/graviton theorems to any dimension using the CHY formulae.

Scattering Equations

external momentum

$$\sum_{i \neq j} \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0$$

point on 2-sphere



- Gross/Mende: These equations arise from the tensionless limit of string amplitudes
- Cachazo/He/Yuan (CHY): They also arise in the amplitudes of massless point particles!

CHY Formulae

- YM:

$$A_n = \frac{1}{\mathrm{vol}\,\mathrm{SL}(2,\mathbb{C})} \int \frac{d^{\,n}\sigma}{\sigma_{12}\ldots\sigma_{n1}} \prod_a{}' \delta(\sum_{b\neq a} \frac{k_a\cdot k_b}{\sigma_{ab}})\,\mathrm{Pf}'\Psi.$$

- Gravity:

$$M_n = \frac{1}{\mathrm{vol}\,\mathrm{SL}(2,\mathbb{C})} \int d^{\,n}\sigma \prod_a{}' \delta(\sum_{b\neq a} \frac{k_a\cdot k_b}{\sigma_{ab}})\,\mathrm{Pf}'\Psi\,\mathrm{Pf}'\tilde{\Psi}$$

where

$$\prod_a{}'\delta(\sum_{b\neq a}\frac{k_a\cdot k_b}{\sigma_{ab}}) \equiv \sigma_{ij}\sigma_{jk}\sigma_{ki}\prod_{a\neq i,j,k}\delta(\sum_{b\neq a}\frac{k_a\cdot k_b}{\sigma_{ab}})$$

and

$$\Psi = \begin{pmatrix} A & -C^{\mathrm{T}} \\ C & B \end{pmatrix}$$

$$A_{ab} = \begin{cases} \dfrac{k_a\cdot k_b}{\sigma_a-\sigma_b} & a\neq b, \\ 0 & a=b, \end{cases} \qquad B_{ab} = \begin{cases} \dfrac{\epsilon_a\cdot \epsilon_b}{\sigma_a-\sigma_b} & a\neq b, \\ 0 & a=b \end{cases}$$

$$C_{ab} = \begin{cases} \dfrac{\epsilon_a\cdot k_b}{\sigma_a-\sigma_b} & a\neq b, \\ -\displaystyle\sum_{c\neq a}\dfrac{\epsilon_a\cdot k_c}{\sigma_a-\sigma_c} & a=b. \end{cases}$$

Ambitwistor Strings

- Mason/Skinner: Amplitudes of complexified massless point particles can be computed using a chiral, infinite tension limit of the RNS string:

$$S = \int_\Sigma P_\mu \bar{\partial} X^\mu + \frac{e}{2} P_\mu P^\mu + \ldots$$

- Correlation functions of vertex operators reproduce the CHY formulae!

- Critical in d=26 (bosonic) and d=10 (superstring)

Ambitwistor Strings and Soft Limits

- Ambitwistor string theory makes the relation between BMS symmetry and soft limits transparent, and implies extensions to gravity and Yang-Mills in arbitrary dimensions.

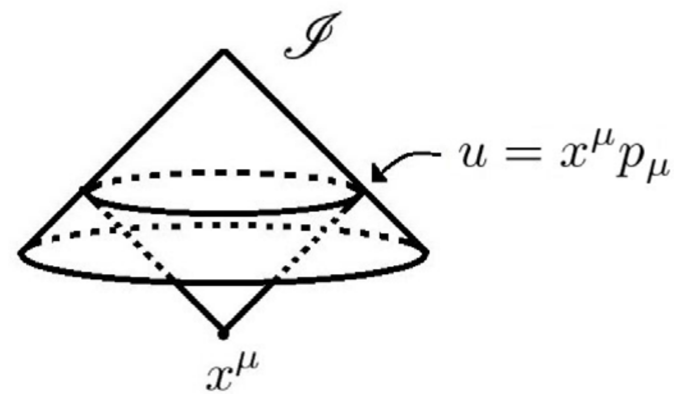
- This approach was inspired by Adamo/Casali/Skinner, who derived the soft limits theorems using a 2d CFT at null infinity.

- A closely related model is the 4d ambitwistor string, which is genuinely twistorial and gives rise to new formulae with any amount of susy (Geyer/Lipstein/Mason).

Ambitwistor Space vs Null Infinity

- A null geodesic through the point x^μ with tangent vector P_μ reaches null infinity at

$$(u, p_\mu) = w(x^\nu P_\nu, P_\mu)$$



- Ambitwistor space can be described using coordinates

$$(u, p_\mu, w, q^\mu)$$

where $q^\mu = w x^\mu$

Ambitwistor Strings at Null Infinity

- Action:

$$S = \frac{1}{2\pi}\int_\Sigma w\bar\partial u - q^\mu \bar\partial p_\mu + \Psi_r \cdot \bar\partial \Psi_r + \\ + eT + \tilde e p^2 + \chi_r w p \cdot \Psi_r + a(uw - q\cdot p)$$

- Integrated vertex operators (gravity):

$$\mathcal{V} \;=\; \int_\Sigma \bar\delta(k\cdot p)\, w\, \mathrm{e}^{ik\cdot q/w} \prod_{r=1}^{2} \epsilon_{r\mu}(p^\mu + i\Psi_r^\mu \Psi_r \cdot k)$$

- Integrated vertex operators (YM):

$$\mathcal{V} = \int_\Sigma \bar\delta(k\cdot p) w \mathrm{e}^{ik\cdot q/w} \epsilon_\mu (p^\mu + i\Psi^\mu \Psi\cdot k) j\cdot t$$

Correlation Functions

- Amplitudes correspond to correlation functions:

$$\mathcal{A}_n = \langle \mathcal{V}_1 ... \mathcal{V}_n \rangle$$

- Combining exponentials with action gives:

$$\mathcal{L} = \frac{1}{2\pi}\left(u\bar{\partial}w - q^\mu \bar{\partial}p_\mu\right) + i\sum_{i=1}^{n} k_i \cdot q/w\,\delta^2\left(\sigma - \sigma_i\right)$$

- u eom: $\bar{\partial}w = 0 \rightarrow w = 1$

- q eom: $\bar{\partial}p = 2\pi i \sum_{i=1}^{n} k_i \delta^2\left(\sigma - \sigma_i\right) \rightarrow \boxed{p(\sigma) = \sum_{i=1}^{n} \frac{k_i}{\sigma - \sigma_i}}$

BMS Symmetry in Ambitwistor Space

- Diffeomorphisms of null infinity lift to Hamiltonian actions of ambitwistor space.

- Translations: $\delta x^\mu = a^\mu$

$$H_a = w a \cdot p \longrightarrow H_f = w f(p)$$ "supertranslation"

where $f(\alpha p) = \alpha f(p)$

- Rotations: $\delta x^\mu = r^\mu{}_\nu x^\nu$, $r_{\mu\nu} = - r_{\nu\mu}$

$$H_r = q^{[\mu} p^{\nu]} r_{\mu\nu} \longrightarrow H_r = q^{[\mu} p^{\nu]} r_{\mu\nu}(p)$$ "superrotation"

where $r_{\mu\nu}(\alpha p) = r_{\mu\nu}(p)$

From Soft Limits to BMS

- Key idea: BMS generators correspond to leading and subleading terms in the Taylor expansion of soft graviton vertex operators.

- To see this, rewrite the graviton vertex operator as follows:

$$\begin{aligned}\mathcal{V} &= \int_\Sigma \bar{\delta}(k\cdot p)\, w\, \mathrm{e}^{ik\cdot q/w} \prod_{r=1}^{2} \epsilon_{r\mu}(p^\mu + i\Psi_r^\mu \Psi_r \cdot k)\,, \\ &= \frac{1}{2\pi i} \oint \frac{\mathrm{e}^{ik\cdot q/w}}{k\cdot p}\, w \prod_{r=1}^{2} \epsilon_{r\mu}(p^\mu + i\Psi_r^\mu \Psi_r \cdot k)\,,\end{aligned}$$

where we noted that $\bar{\delta}(k\cdot p) = \frac{1}{2\pi i}\bar{\partial}\frac{1}{k\cdot p}$

- Taylor expanding in the soft momentum s then gives

$$\mathcal{V}_s \;=\; \mathcal{V}_s^0 + \mathcal{V}_s^1 + \mathcal{V}_s^2 + \mathcal{V}_s^3 + \ldots .$$

where

$$\mathcal{V}_s^0 = \frac{1}{2\pi i}\oint w\frac{(\epsilon\cdot p)^2}{s\cdot p} \qquad \mathcal{V}_s^1 = \frac{1}{2\pi}\oint \frac{\epsilon\cdot p}{s\cdot p}\epsilon^\mu s^\nu J_{\mu\nu}$$

and $J_{\mu\nu} = p_{[\mu}q_{\nu]} + w\sum_{r=1}^{2}\Psi_{r\mu}\Psi_{r\nu}$

- Note that the leading(subleading) term is a supertranslation (superrotation) generator!

From BMS to Soft Limits

- Insertion of supertranslation generator:

$$\left\langle \mathcal{V}_1 \ldots \mathcal{V}_n \mathcal{V}_s^0 \right\rangle = \left(\sum_{a=1}^{n} \frac{(\epsilon \cdot k_a)^2}{s \cdot k_a} \right) \langle \mathcal{V}_1 \ldots \mathcal{V}_n \rangle$$

- Insertion of superrotation generator:

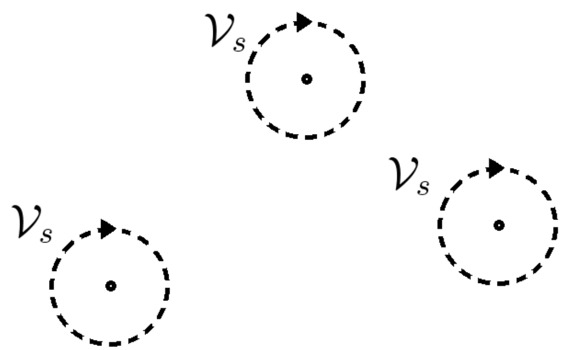
$$\left\langle \mathcal{V}_1 \ldots \mathcal{V}_n \mathcal{V}_s^1 \right\rangle = \sum_{a=1}^{n} \frac{\epsilon_{\mu\nu} k_a^{\mu} s_{\lambda} J_a^{\lambda\nu}}{s \cdot k_a} \langle \mathcal{V}_1 \ldots \mathcal{V}_n \rangle$$

where $\epsilon^{\mu\nu} = \epsilon^{\mu}\epsilon^{\nu}$ and $J_a^{\mu\nu} = k_a^{[\mu} \frac{\partial}{\partial k_{a,\nu]}} + \epsilon_a^{[\mu} \frac{\partial}{\partial \epsilon_{a,\nu]}}$

- To see this, consider a soft graviton insertion:

$$\langle \mathcal{V}_1 \ldots \mathcal{V}_n \mathcal{V}_s \rangle$$

- This can be computed by integrating the soft vertex operator around the hard ones and adding up the residues:



- For leading and subleading soft terms, these residues do not depend on the detailed structure of the hard vertex operators, reflecting universality.

Analogue for YM

- Insertion of "supertranslation" generator:

$$\left\langle \mathcal{V}_1 \ldots \mathcal{V}_n \mathcal{V}_s^{ym,0} \right\rangle = \left(\frac{\epsilon \cdot k_1}{s \cdot k_1} - \frac{\epsilon \cdot k_n}{s \cdot k_n} \right) \langle \mathcal{V}_1 \ldots \mathcal{V}_n \rangle$$

- Insertion of "superrotation" generator:

$$\left\langle \mathcal{V}_1 \ldots \mathcal{V}_n \mathcal{V}_s^{ym,1} \right\rangle = \left(\frac{\epsilon_\mu s_\nu J_1^{\mu\nu}}{s \cdot k_1} - \frac{\epsilon_\mu s_\nu J_n^{\mu\nu}}{s \cdot k_n} \right) \langle \mathcal{V}_1 \ldots \mathcal{V}_n \rangle$$

Summary

- Complexified massless point-particles can be formulated as ambitwistor strings.

- Ambitwistor string theory provides new insight into BMS symmetries and their relationship to soft limits.

- In particular, BMS generators correspond to leading and subleading terms in the expansion of soft graviton vertex operators, and there is a similar story for YM.

- Higher order terms generate diffeomorphisms of ambitwistors space, but not null infinity.

Open Questions

- The leading and subleading terms of soft graviton vertex operators appear to generate an infinite dimensional algebra. What is this algebra general dimensions?
- What is the explicit field theory representation of higher order soft limits?
- What is the fate of BMS symmetry at loop level from the point of view of ambitwistor string theory?

Thank You