New geometric structures

in scattering amplitudes

The anatomy of scattering amplitudes

in pure spinor superspace

Oliver Schlotterer (AEI Potsdam)

based on arXiv:1404.4986, arXiv:1408.3605: C. Mafra, OS

and work in progress with M. Green and C. Mafra

22.09.2014

Goal of this talk

- framework for amplitudes of gluon and graviton multiplet in 10 dim
- both field theory and string theory, both type IIA and type IIB
- manifest supersymmetry from pure spinor formalism

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Intuitive mapping between

- cubic diagrams and kinematic factors
- kinematic factors and worldsheet functions

Make essential use of BRST symmetry in the pure spinor formalism

[N. Berkovits hep-th/0001035]

Pure spinor superspace

Bosonic pure spinor λ^α defined by algebraic constraint

$$\lambda^\alpha \, \gamma^m_{\alpha\beta} \, \lambda^\beta \quad = \quad 0 \quad \forall \;\; m = 0, 1, \ldots, 9$$

Pure spinor superspace (PSS) $\{x^m, \; \theta^\alpha, \; \lambda^\beta\}$ with component prescription

$$\langle \, (\lambda\gamma^m\theta) \, (\lambda\gamma^n\theta) \, (\lambda\gamma^p\theta) \, (\theta\gamma_{mnp}\theta) \, \rangle \quad = \quad 1$$

BRST invariant & supersymmetric and automated in **[C. Mafra 1007.4999]**

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Driving force towards amplitudes in PSS: BRST charge $\leftrightarrow$ eq's of motion

$$Q \quad \equiv \quad \lambda^\alpha D_\alpha \quad = \quad \lambda^\alpha \left(\frac{\partial}{\partial \theta^\alpha} + \tfrac{1}{2} \, k_m \, (\gamma^m \theta)_\alpha \right)$$

descends from gauge fixing worldsheet action **[see Nathan's talk and 1409.2510]**

Outline

scattering amplitudes

gluon & gluino polariz. e^m, χ^α

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10 dim $\mathcal{N} = 1$ SYM superfields

gluon & gluino polariz. e^m, χ^α

$$A_\alpha(x,\theta) \;=\; e^{ik\cdot x}\Big(\tfrac{1}{2}\, e_m\, (\gamma^m\theta)_\alpha - \tfrac{1}{3}(\chi\gamma_m\theta)(\gamma^m\theta)_\alpha + \theta^3 ke + \theta^4\chi k + \theta^5 k^2 e + \ldots\Big)$$

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multiparticle superfields

part I based on 1404.4986

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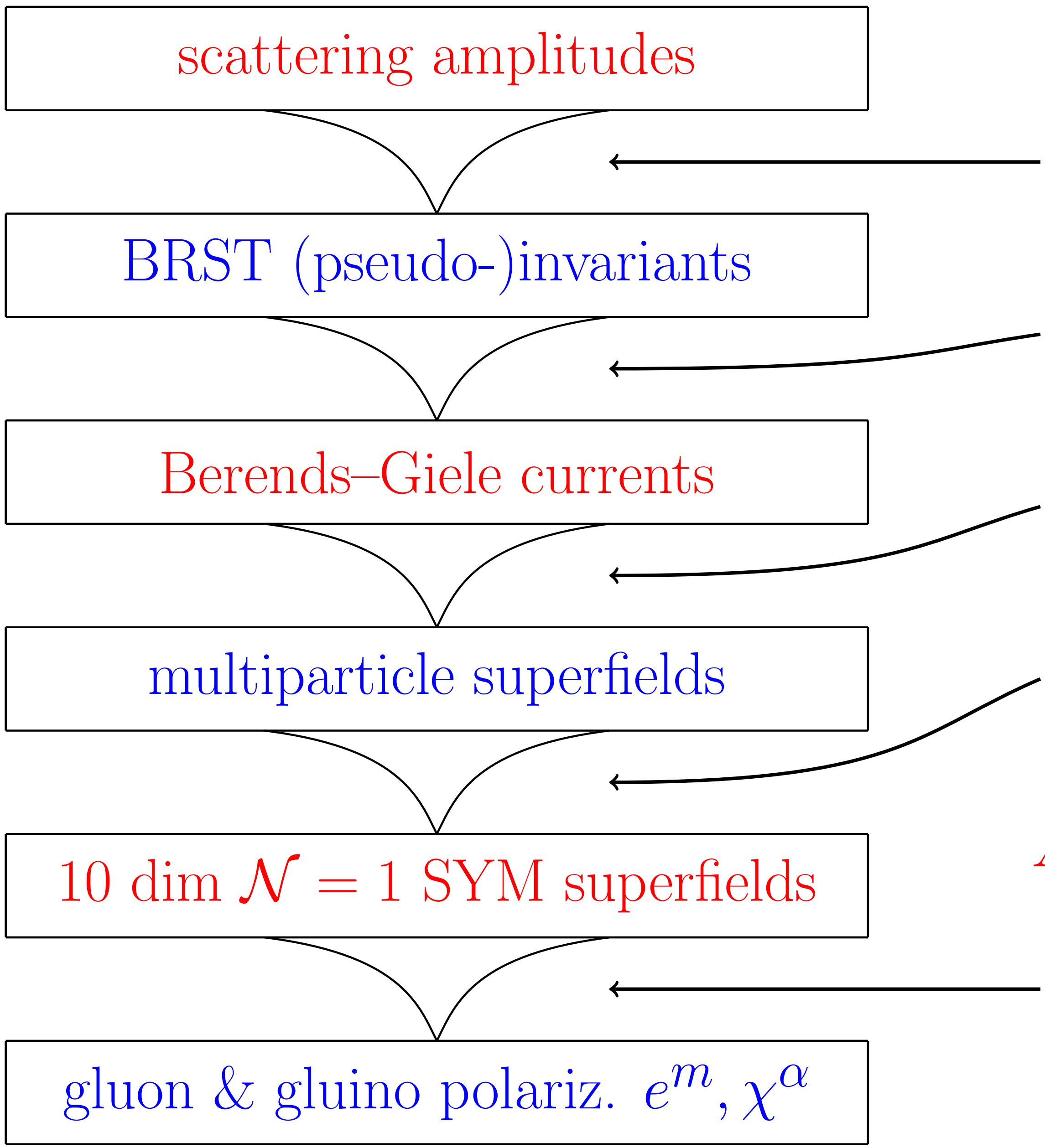
Outline



part IV based on [to appear]

part III based on 1408.3605

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I. Multiparticle superfields

Vertex operators for SYM states (unintegrated and integrated)

$$V_1 \equiv \lambda^\alpha A^1_\alpha \, , \qquad U_1 \equiv \partial\theta^\alpha A^1_\alpha + \Pi_m A^m_1 + d_\alpha W^\alpha_1 + \frac{1}{2} N_{mn} F^{mn}_1$$

superfields with known θ expansion $\otimes$ $h = 1$ fields as “bookkeeping var’s”

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BRST invariance $QV_1 = 0$ and $Q \int U_1 = \int \frac{\partial}{\partial z} V_1 = 0$ equivalent to

equations of motion (since $Q = \lambda^\alpha D_\alpha$ on superfields) **[E. Witten 1986]**

$$2D_{(\alpha}A^1_{\beta)} = \gamma^m_{\alpha\beta} A^1_m$$

$$D_\alpha A^1_m = (\gamma_m W_1)_\alpha + k^1_m A^1_\alpha$$

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$$D_\alpha W^\beta_1 = \frac{1}{4}(\gamma_{mn})_\alpha{}^\beta F^{mn}_1 \qquad k_m e^m = 0$$

$$D_\alpha F^{mn}_1 = 2k^{[m}_1 (\gamma^{n]} W_1)_\alpha \qquad \not{k}_{\alpha\beta}\chi^\beta = 0$$

Define multiparticle superfields via OPE (where $z_{ij} \equiv z_i - z_j$)

$$U_1(z_1)U_2(z_2) \sim z_{12}^{\alpha' k_1 \cdot k_2 - 1}\Big(\partial\theta^\alpha A_\alpha^{12} + \Pi_m A_{12}^m + d_\alpha W_{12}^\alpha + \frac{1}{2} N_{mn} F_{12}^{mn}\Big) + \frac{\partial}{\partial z_i}(\ldots)$$

Same structure as $U_1 = \partial\theta^\alpha A_\alpha^1 + \Pi_m A_1^m + d_\alpha W_1^\alpha + \frac{1}{2} N_{mn} F_1^{mn}$ with

$$A_\alpha^{12} = \frac{1}{2}\big[A_\alpha^2 (k^2 \cdot A^1) + A_m^2 (\gamma^m W^1)_\alpha - (1 \leftrightarrow 2)\big]$$

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$$W^\alpha_{12} = \tfrac{1}{4}(\gamma^{mn}W^2)^\alpha F^1_{mn} + W^\alpha_2(k^2\cdot A^1) - (1\leftrightarrow 2)$$

$$F^{12}_{mn} = F^2_{mn}(k^2\cdot A^1) + F^2_{[m}{}^p F^1_{n]p} + 2k^1_{[m}(W^1\gamma_{n]}W^2) - (1\leftrightarrow 2)$$

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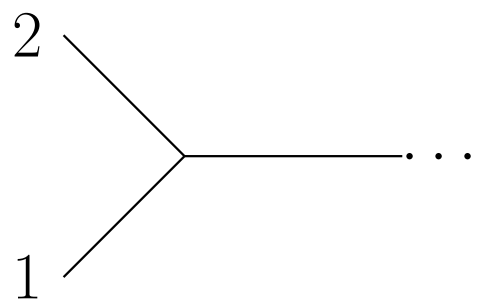
$\Downarrow$

Four superfield rep's

of the cubic vertex



$\leftrightarrow$ $A^{12}_\alpha,\ A^m_{12},\ W^\alpha_{12},\ F^{mn}_{12}$

$$V_{12} \equiv \lambda^\alpha A^{12}_\alpha\,, \qquad U_{12} \equiv \partial\theta^\alpha A^{12}_\alpha + \Pi_m A^m_{12} + d_\alpha W^\alpha_{12} + \frac{1}{2} N_{mn} F^{mn}_{12}$$

Two-particle EOM $\cong$ single-particle EOM ...

$$\begin{aligned} 2D_{(\alpha} A^1_{\beta)} &= \gamma^m_{\alpha\beta} A^1_m \\ D_\alpha A^1_m &= (\gamma_m W_1\ )_\alpha + k^1_m A^1_\alpha \\ D_\alpha W^\beta_1 &= \frac{1}{4} (\gamma_{mn})_\alpha{}^\beta F^{mn}_1 \\ D_\alpha F^{mn}_1 &= 2k^{[m}_1 (\gamma^{n]} W_1\ )_\alpha \end{aligned}$$

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Two-particle EOM $\cong$ single-particle EOM up to contact terms $\sim (k_1 \cdot k_2)$

$$2D_{(\alpha} A^{12}_{\beta)} = \gamma^m_{\alpha\beta} A^{12}_m + (k_1 \cdot k_2)(A^1_\alpha A^2_\beta - A^2_\alpha A^1_\beta)$$

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$$+ 2(k_1 \cdot k_2)(A^{[n}_1 (\gamma^{m]} W_2)_\alpha - A^{[n}_2 (\gamma^{m]} W_1)_\alpha)$$

where $k^m_{12} \equiv k^m_1 + k^m_2$

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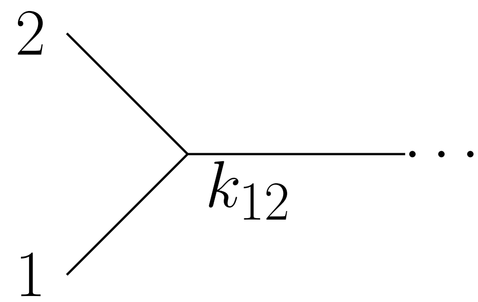
BRST invariance $QV_1 = 0$ and $Q \int U_1 = \int \frac{\partial}{\partial z} V_1$ replaced by covariance

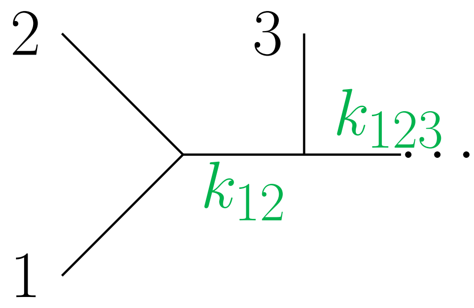
$$QV_{12} = (k_1 \cdot k_2) V_1 V_2 \,, \qquad QU_{12} = \frac{\partial}{\partial z} V_{12} + (k_1 \cdot k_2)(V_1 U_2 - V_2 U_1)$$

More particles by recursion



$$V_{12} = \frac{1}{2}\left[V_2(k^2 \cdot A^1\ ) + A^2_m(\lambda\gamma^m W_1\ ) - (1\ \leftrightarrow 2)\right]$$

More particles by recursion (replacing [1,2] by [12,3])



$$\widehat{V}_{123} = \frac{1}{2}\left[V_3(k^3 \cdot A^{12}) + A^3_m(\lambda\gamma^m W_{12}) - (12 \leftrightarrow 3)\right]$$

BRST variation cancels propagators $\sim k^2_{12}, k^2_{123}$ of the cubic diagram

$$Q\widehat{V}_{123} \;=\; \frac{1}{2}(k^2_{123} - k^2_{12})V_{12}V_3 + \frac{1}{2}k^2_{12}(V_1V_{23} - V_2V_{13})$$

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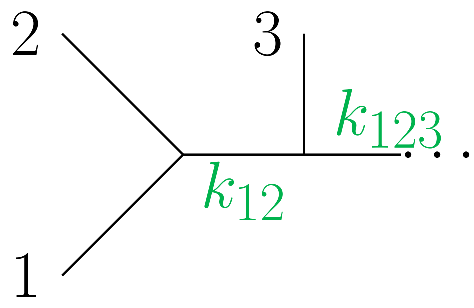

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Moreover – totally antisymmetric component is BRST closed and exact

$$V_{123} \;=\; \widehat{V}_{123} + QH_{[123]} \quad \Longrightarrow \quad V_{123} + V_{231} + V_{312} \;=\; 0$$

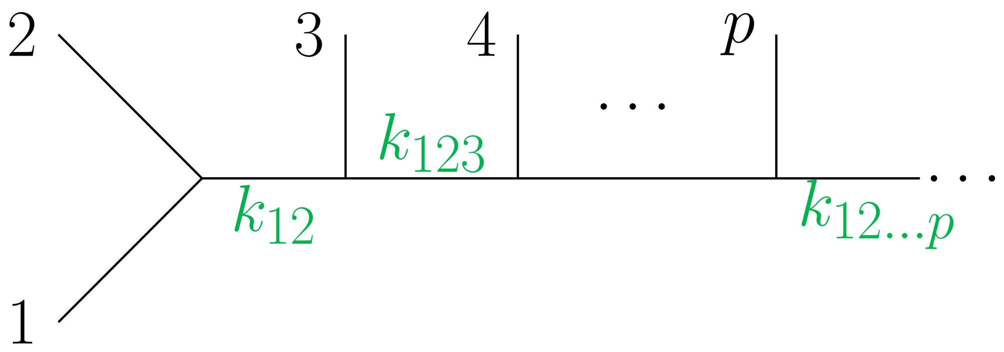
Reproduce Jacobi identity among color tensors $f^{12a}f^{a3b} + \text{cyc}(1,2,3) = 0$

$\Longrightarrow$ evidence for duality between color and kinematics

[Bern, Carrasco, Johansson 0805.3993]

[Mafra, OS, Stieberger 1104.5224]

In multiparticle cases $B = 12\ldots p\ \exists\ 4$ representatives of cubic tree diag's



$$\leftrightarrow \quad A_\alpha^B,\ A_B^m,\ W_B^\alpha,\ F_B^{mn}$$

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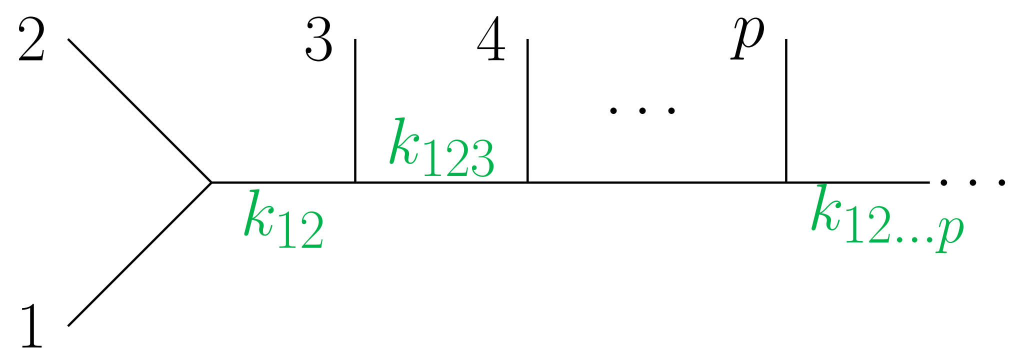

$\leftrightarrow \quad A^B_\alpha,\ A^m_B,\ W^\alpha_B,\ F^{mn}_B$

with EOM of $A^1_\alpha,\ A^m_1,\ W^\alpha_1,\ F^{mn}_1$ up to contact terms $\sim k^2_{12\dots j}$

- iterate recursions, e.g. for $\widehat{V}_B = \lambda^\alpha \widehat{A}^B_\alpha$

$$\widehat{V}_{12\dots p} = \frac{1}{2}\big[V_p(k^p \cdot A^{12\dots p-1}) + A^p_m(\lambda\gamma^m W_{12\dots p-1}) - (12\dots p-1 \leftrightarrow p)\big]$$

- discard BRST trivial components $V_B = \widehat{V}_B + Q(\dots)$ "by hand"

$\Longrightarrow$ symmetries of dual color tensor $f^{12a}f^{a3b}f^{b4c}\dots f^{ypz}$

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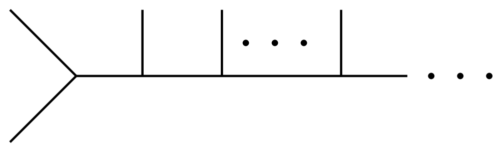
- scattering amplitudes
- BRST (pseudo-)invariants
- Berends–Giele currents
- multiparticle superfields
- 10 dim $\mathcal{N}=1$ SYM superfields
- gluon & gluino polariz. e^m, χ^α

multiparticle superfields $\longleftarrow$ $A^B_\alpha,\ A^m_B,\ W^\alpha_B,\ F^{mn}_B$



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II. Berends–Giele currents

Motivation: Package tree level subdiagrams and clean up BRST variation

$$QV_{123} = (s_{123} - s_{12})V_{12}V_3 + s_{12}(V_1V_{23} - V_2V_{13}) \quad \text{with} \quad s_{ij} = (k_i \cdot k_j)$$

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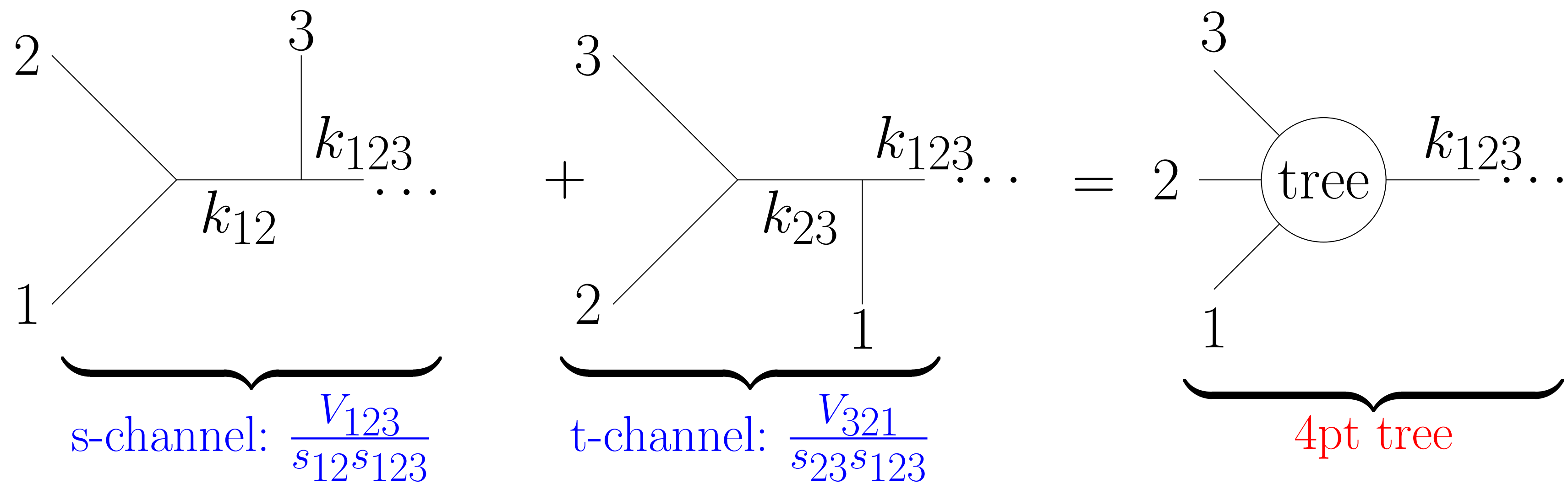
Both can be achieved by assembling color ordered tree with off-shell leg

M_{123} =



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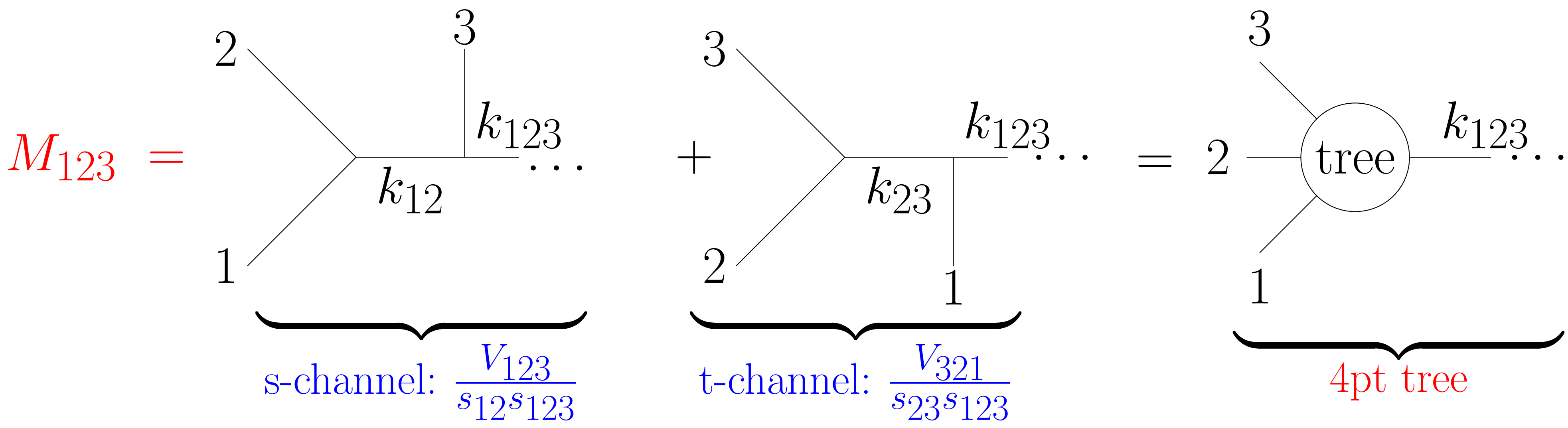
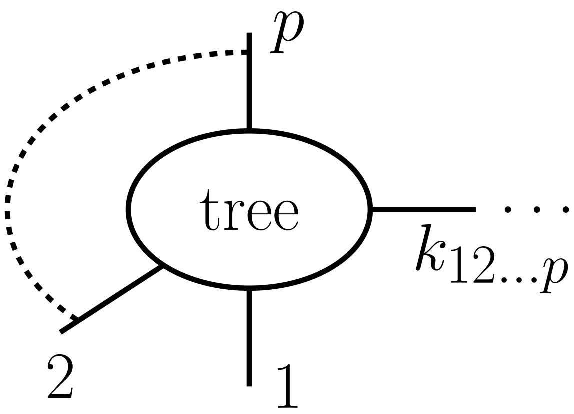


Reward: Q action simply deconcatenates (with $M_{12} \equiv \frac{V_{12}}{s_{12}}$ and $M_1 = V_1$)

$$QM_{123} = M_{12}M_3 + M_1M_{23}$$

p-particle BG current $M_{12\ldots p} \;\leftrightarrow\; (p+1)$-point tree (color ordered):

$M_{12\ldots p} \quad \longleftrightarrow$



$$= \sum_{\text{cubic diags}} \left[\frac{V_{i_1 i_2 \ldots i_p}}{(s_{kl})^{p-1}} \right]$$

Mandelstams s_{kl} in $QV_{12\ldots p}$ replaced by deconcatenation

$$QM_{12\ldots p} = \sum_{j=1}^{p-1} M_{12\ldots j} M_{j+1\ldots p} = \sum_{XY=12\ldots p} M_X M_Y$$

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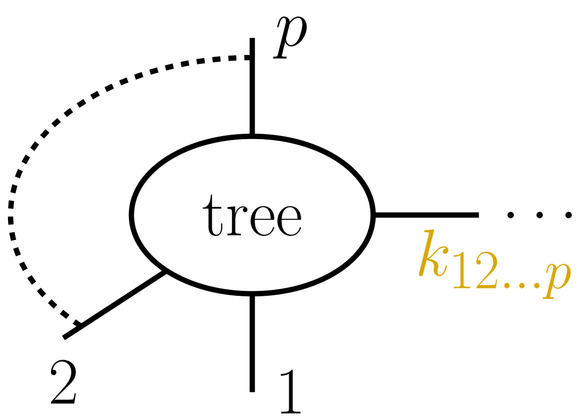

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$$QM_{12\ldots p} = \sum_{j=1}^{p-1} M_{12\ldots j} M_{j+1\ldots p} = \sum_{XY=12\ldots p} M_X M_Y$$

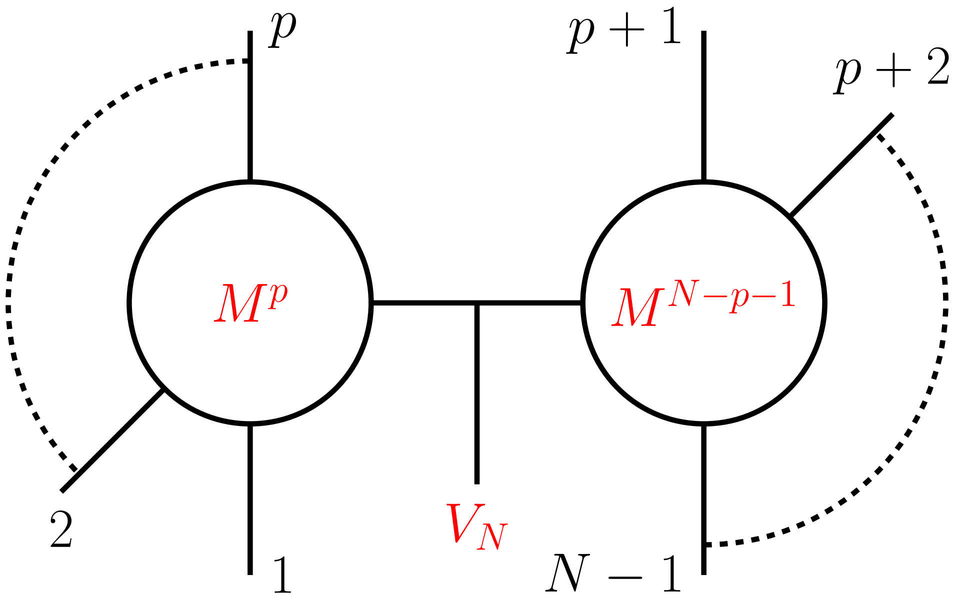
Cancels overall propagator:

$$M_{12\ldots p} \sim \frac{1}{s_{12\ldots p}} \,, \quad QM_{12\ldots p} \text{ regular in } s_{12\ldots p}$$

Application: color ordered SYM trees

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,N) \;=\; \sum_{p=1}^{N-2} \langle\, M_{12\ldots p}\, M_{p+1\ldots N-1}\, V_N \,\rangle$$

$$= \sum_{p=1}^{N-2}$$



fixed by pole structure

and BRST invariance !

[Mafra, OS, Stieberger, Tsimpis 1012.3981]

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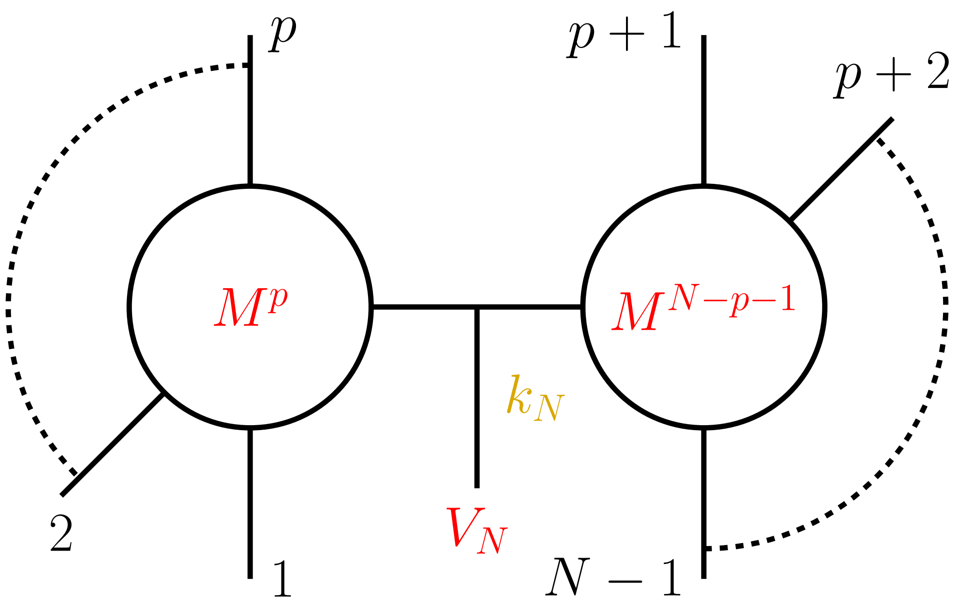


fixed by pole structure and BRST invariance !

[Mafra, OS, Stieberger, Tsimpis 1012.3981]

- divergent propagator $M_{12\ldots N-1} \sim \frac{1}{s_{12\ldots N-1}} \sim \frac{1}{k_N^2} \to \infty$

  avoids Q-exactness "$\mathcal{A}^{\text{tree}}(1,2,\ldots,N) \;=\; \langle Q(M_{12\ldots N-1} V_N)\rangle$"

- components fully explicit after $\langle \lambda^3 \theta^5 \rangle = 1$

$\exists$ further BG currents $\mathcal{A}_B^m, \mathcal{W}_B^\alpha, \mathcal{F}_B^{mn}$ for $A_B^m, W_B^\alpha, F_B^{mn}$ with $B = 12\ldots p$

$$\mathcal{A}_{123}^m = \frac{A_{123}^m}{s_{12}s_{123}} + \frac{A_{321}^m}{s_{23}s_{123}}$$

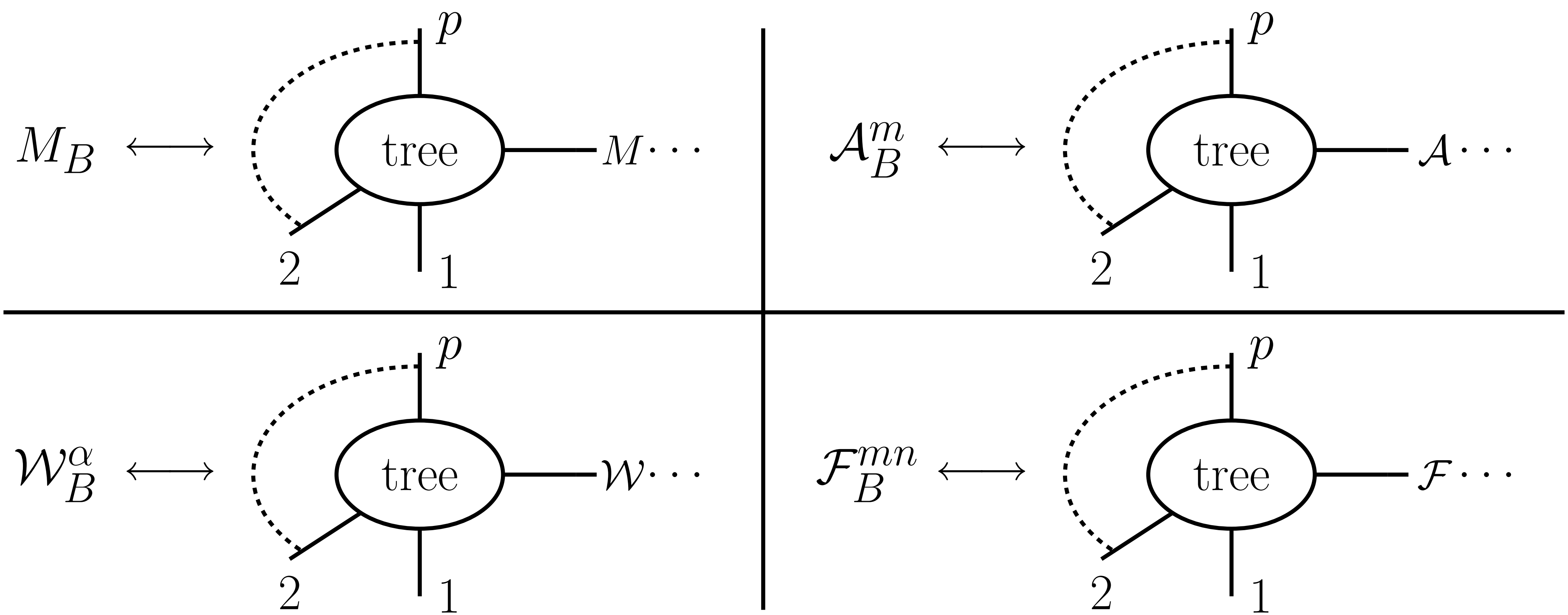
"just a change of basis"

$\exists$ further BG currents $\mathcal{A}^m_B, \mathcal{W}^\alpha_B, \mathcal{F}^{mn}_B$ for $A^m_B, W^\alpha_B, F^{mn}_B$ with $B = 12\ldots p$

$$\mathcal{A}^m_{123} = \frac{A^m_{123}}{s_{12}s_{123}} + \frac{A^m_{321}}{s_{23}s_{123}}$$

"just a change of basis"

$\Rightarrow$ 4 representatives of tree subamplitudes with off-shell leg



Infer selection rules from zero mode saturation and BRST symmetry!

Q algebra $\equiv$ universal single-particle EOM plus deconcatenation:

$$QM_B = \sum_{XY=B} M_X M_Y$$

$$Q\mathcal{A}^m_B = (\lambda\gamma^m \mathcal{W}_B) + k^m_B M_B + \sum_{XY=B} (M_X \mathcal{A}^m_Y - M_Y \mathcal{A}^m_X)$$

$$Q\mathcal{W}^\alpha_B = \frac{1}{4}(\lambda\gamma_{mn})^\alpha \mathcal{F}^{mn}_B + \sum_{XY=B} (M_X \mathcal{W}^\alpha_Y - M_Y \mathcal{W}^\alpha_X)$$

$$Q\mathcal{F}^{mn}_B = 2\, k^{[m}_B (\lambda\gamma^{n]} \mathcal{W}_B) + \sum_{XY=B} (M_X \mathcal{F}^{mn}_Y - M_Y \mathcal{F}^{mn}_X)$$

$$+ 2 \sum_{XY=B} (\mathcal{A}^{[n}_X (\lambda\gamma^{m]} \mathcal{W}_Y) - \mathcal{A}^{[n}_Y (\lambda\gamma^{m]} \mathcal{W}_X))$$

Q algebra $\equiv$ universal single-particle EOM plus deconcatenation:

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$$Q\mathcal{W}_B^\alpha = \frac{1}{4}(\lambda\gamma_{mn})^\alpha \mathcal{F}_B^{mn} + \sum_{XY=B} (M_X \mathcal{W}_Y^\alpha - M_Y \mathcal{W}_X^\alpha)$$

$$Q\mathcal{F}_B^{mn} = 2\, k_B^{[m}(\lambda\gamma^{n]}\mathcal{W}_B) + \sum_{XY=B} (M_X \mathcal{F}_Y^{mn} - M_Y \mathcal{F}_X^{mn})$$

$$+ 2 \sum_{XY=B} (\mathcal{A}_X^{[n}(\lambda\gamma^{m]}\mathcal{W}_Y) - \mathcal{A}_Y^{[n}(\lambda\gamma^{m]}\mathcal{W}_X))$$

Simpler as compared to $Q\{A_B^m, W_B^\alpha, F_B^{mn}\} \leftrightarrow (k_{12\ldots j-1} \cdot k_j)$.

Outline

scattering amplitudes

↓

BRST (pseudo-)invariants

↓

Berends–Giele currents

↓

multiparticle superfields

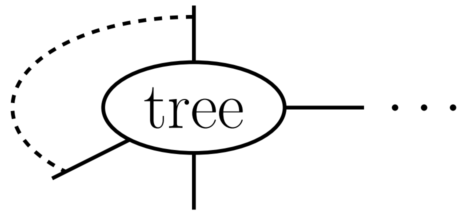
↓

10 dim $\mathcal{N}=1$ SYM superfields

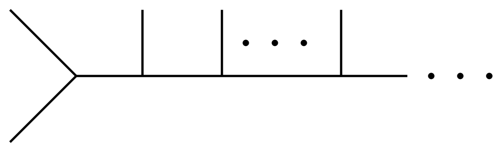
↓

gluon & gluino polariz. e^m, χ^α

$$\left\{ M_B,\ \mathcal{A}_B^m,\ \mathcal{W}_B^\alpha,\ \mathcal{F}_B^{mn} \right.$$



$$\left\{ A_\alpha^B,\ A_B^m,\ W_B^\alpha,\ F_B^{mn} \right.$$



$$A_\alpha(x,\theta) \;=\; e^{ik\cdot x}\Big(\tfrac{1}{2}\,e_m\,(\gamma^m\theta)_\alpha - \tfrac{1}{3}(\chi\gamma_m\theta)(\gamma^m\theta)_\alpha + \theta^3 ke + \theta^4\chi k + \theta^5 k^2 e + \ldots\Big)$$

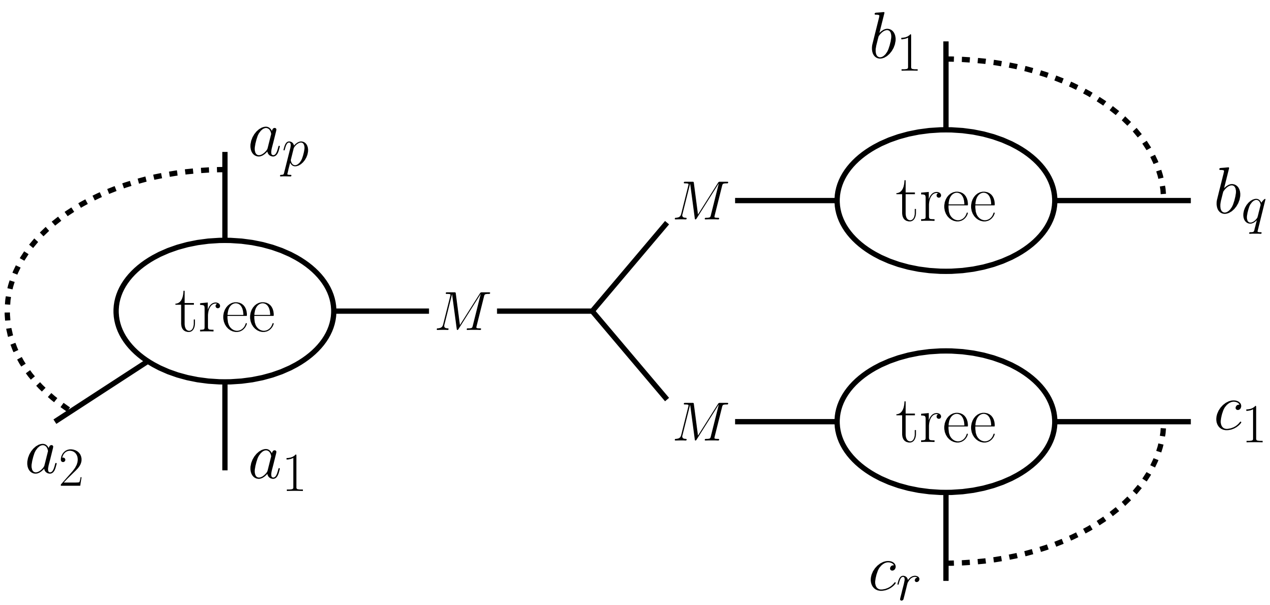
III. BRST (pseudo-)invariants at one-loop

Tree level example of string theory input: disk amplitude prescription

$$\mathcal{A}^{\text{tree}}_{\text{string}}(\alpha') \;=\; \int_{\text{disk}} \langle\, V_1\, V_{N-1}\, V_N\, U_2\, U_3 \,\ldots\, U_{N-2}\,\rangle$$

Iterated OPEs $(U_A U_B \to U_C)$ & $(V_A U_B \to V_C)$ $\Rightarrow$ kinematic pattern

$$\mathcal{A}^{\text{tree}}_{\text{string}}(\alpha') \;\leftrightarrow\; \langle\, M_A\, M_B\, M_C\,\rangle$$



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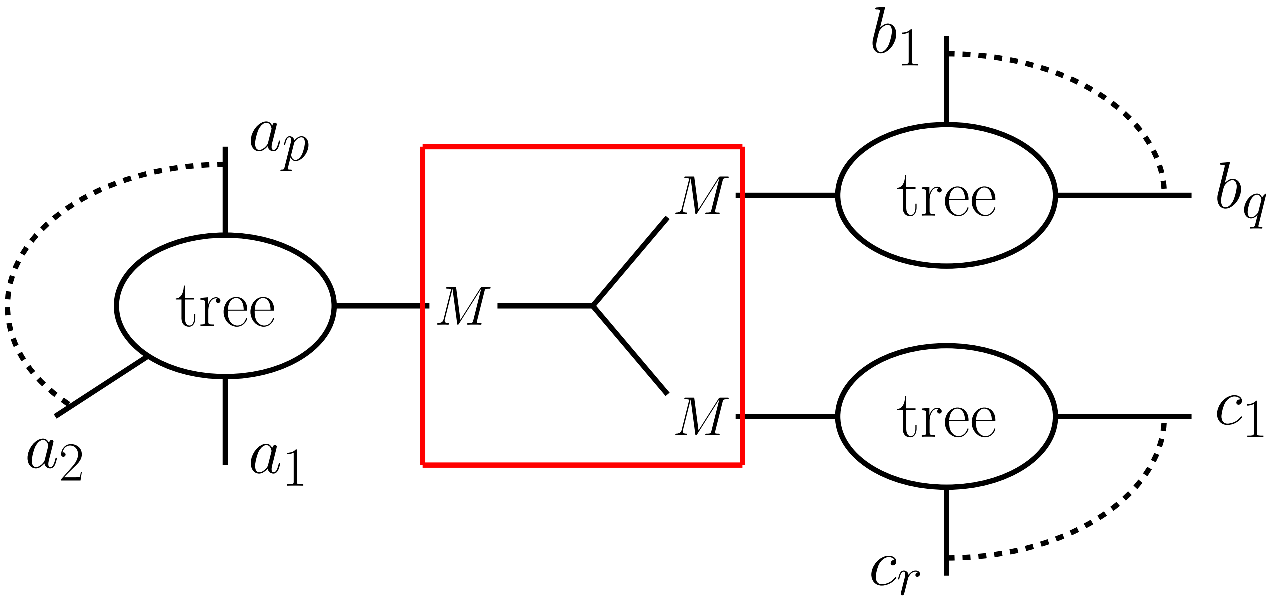

At one-loop: Regulator $\mathcal{N}$ and b-ghost leave 3 zero modes unsaturated

$$\mathcal{A}^{\text{1-loop}}_{\text{string}}(\alpha') \;=\; \int_{\text{cylinder}} \langle\, (\mathcal{N} b) \;\times\; V_1 \; \underbrace{U_2\, U_3\, U_4 \,\ldots\, U_{N-1}\, U_N}_{\text{need zero modes } d_\alpha d_\beta N^{mn}} \rangle$$

OPEs among U_i build up multiparticle vertices

$$\mathcal{U}_B \;\equiv\; \partial\theta^\alpha \mathcal{A}^B_\alpha + \Pi_m \mathcal{A}^m_B + d_\alpha \mathcal{W}^\alpha_B + \tfrac{1}{2} N_{mn} \mathcal{F}^{mn}_B$$

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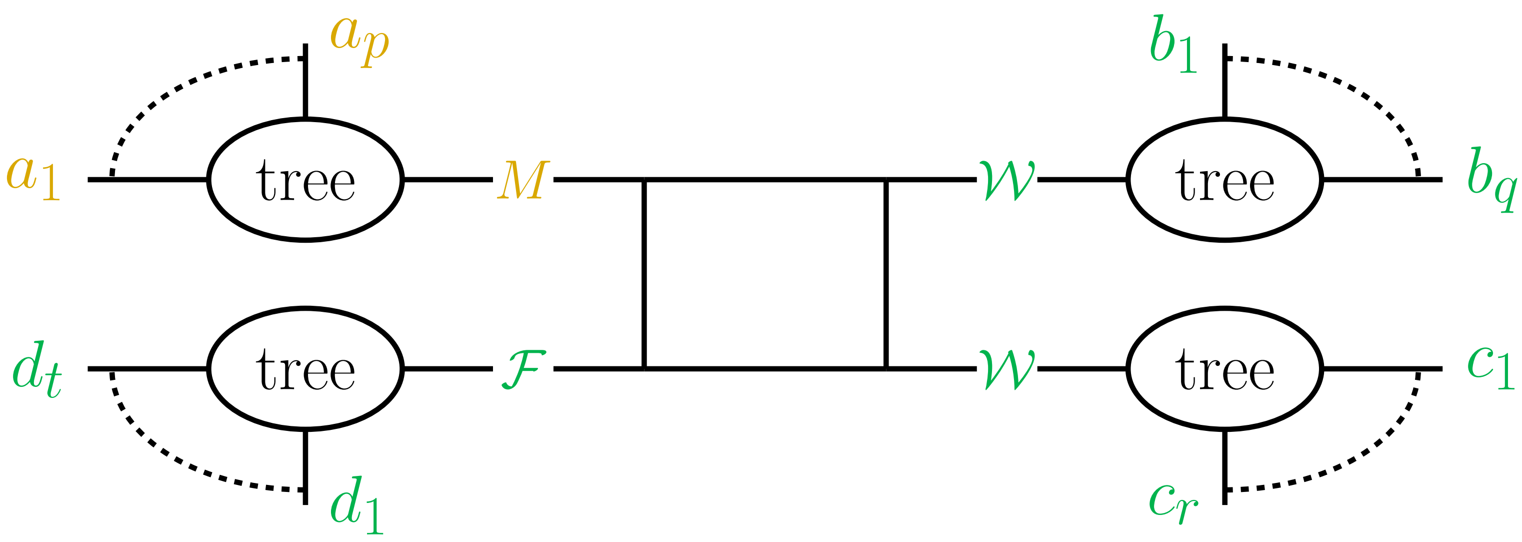
Extraction of $d_\alpha d_\beta N^{mn}$ $\Rightarrow$ kinematic pattern

$$\mathcal{A}^{\text{1-loop}}_{\text{string}}(\alpha') \;\leftrightarrow\; \langle\, M_A \, \mathcal{W}^\alpha_B \, \mathcal{W}^\beta_C \, \mathcal{F}^{mn}_D \,\rangle$$



At one-loop: Regulator $\mathcal{N}$ and b-ghost leave 3 zero modes unsaturated

$$\mathcal{A}^{\text{1-loop}}_{\text{string}}(\alpha') \;=\; \int_{\text{cylinder}} \langle\, (\mathcal{N} b) \times V_1 \underbrace{U_2 U_3 U_4 \ldots U_{N-1} U_N}_{\text{need zero modes } d_\alpha d_\beta N^{mn}} \rangle$$

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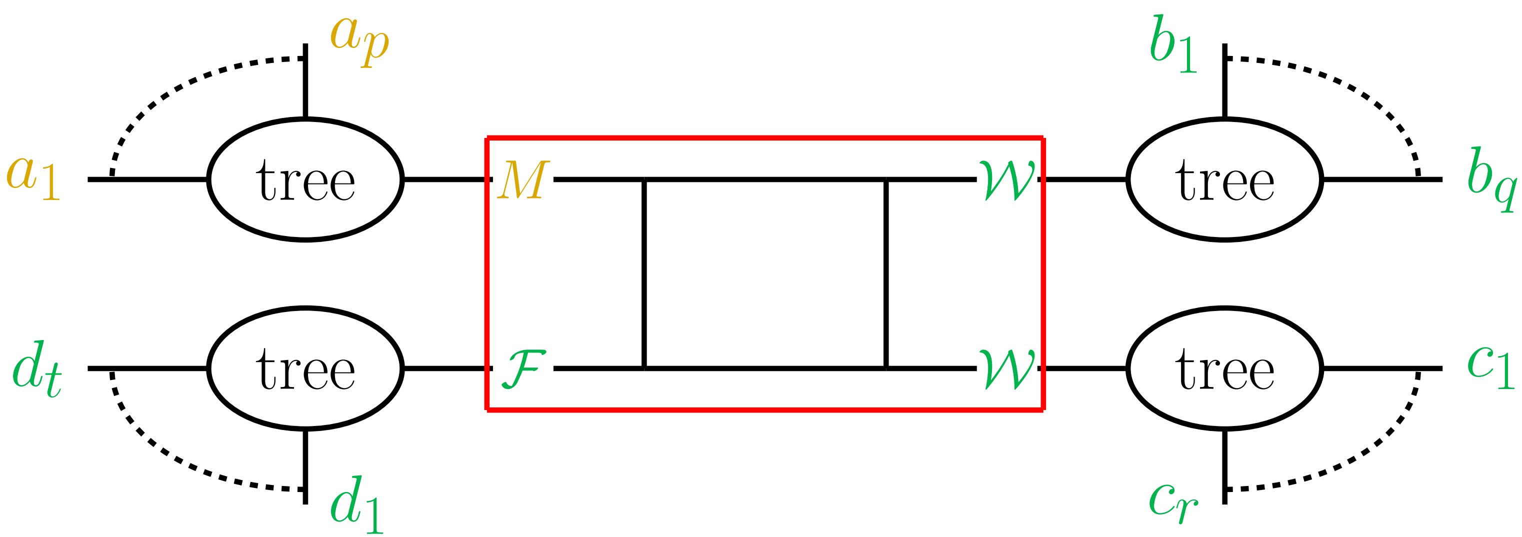
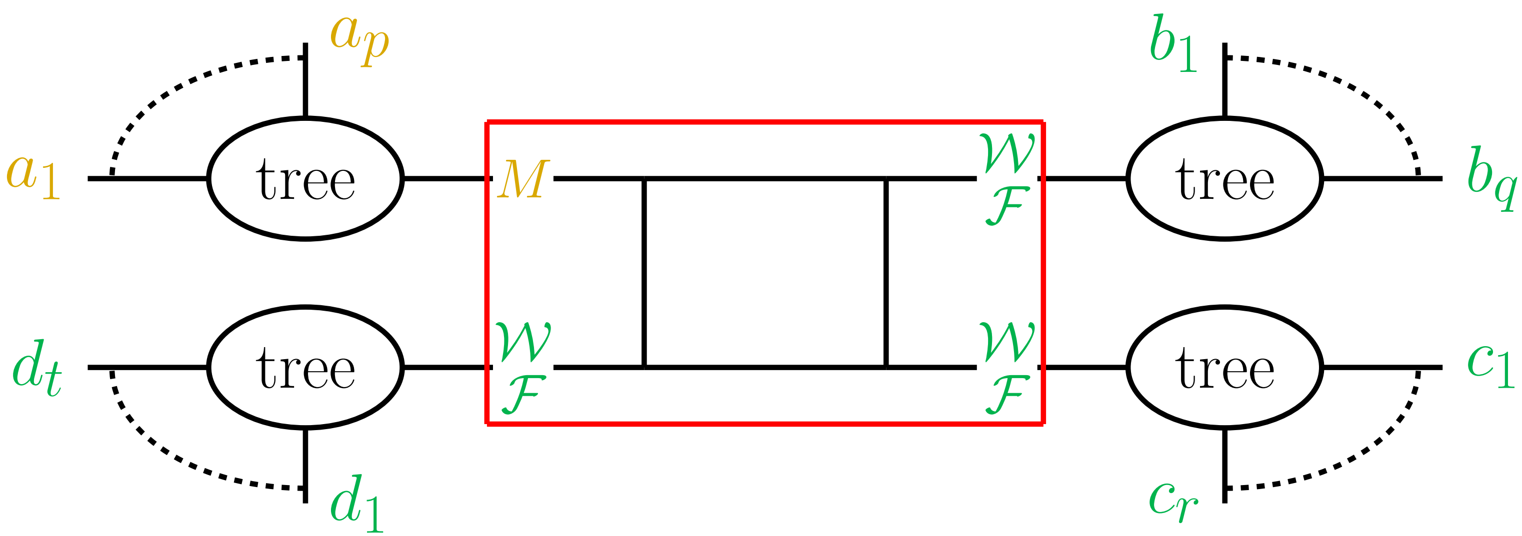


Tensor structure of one-loop kinematic pattern is unique!

$$\mathcal{A}^{\text{1-loop}}_{\text{string}}(\alpha') \quad \leftrightarrow \quad \langle \underbrace{M_A}_{\sim\lambda} \; \underbrace{M_{B,C,D}}_{\text{need } \mathcal{O}(\lambda^2)} \rangle$$

$$M_{B,C,D} \;\equiv\; \tfrac{1}{3}\,(\lambda\gamma_m \mathcal{W}_B)\,(\lambda\gamma_n \mathcal{W}_C)\,\mathcal{F}^{mn}_D + (D \leftrightarrow C, B)$$



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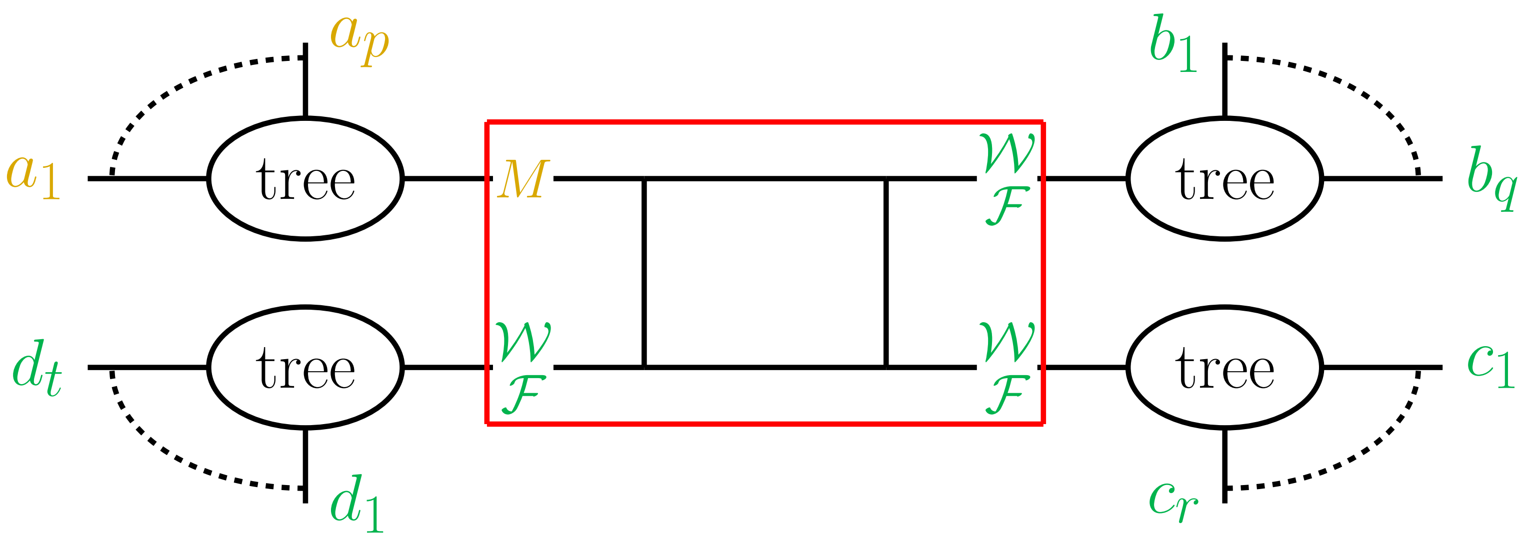


By EOM for $\mathcal{W}^\alpha_B$ and $\mathcal{F}^{mn}_B$: BRST covariance

$$QM_{B,C,D} = \sum_{XY=B} (M_X M_{Y,C,D} - M_Y M_{X,C,D}) + (B \leftrightarrow C, D)$$

BRST covariant building blocks suitable to construct scalar invariants

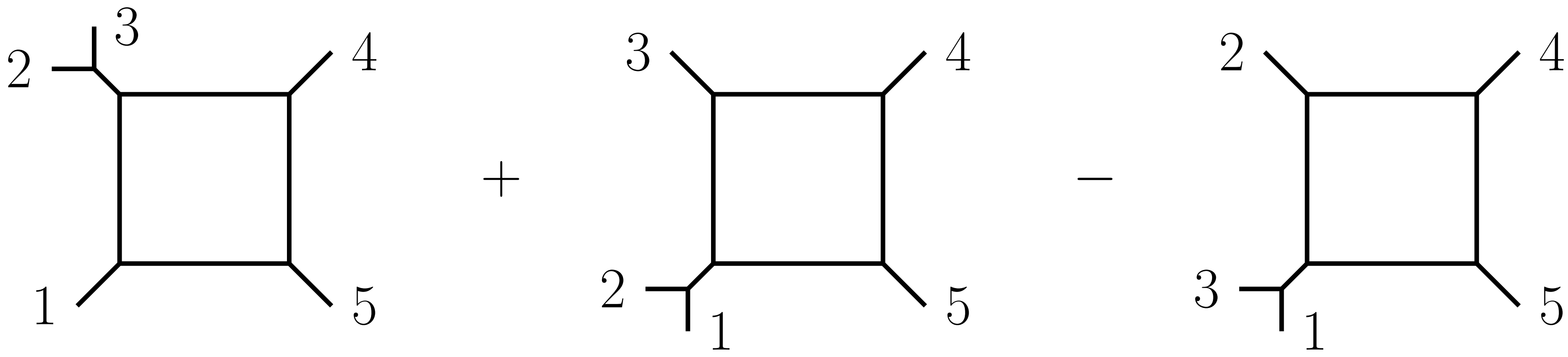
$$QM_A = \sum_{XY=A} M_X M_Y$$

$$QM_{B,C,D} = \sum_{XY=B} (M_X M_{Y,C,D} - M_Y M_{X,C,D}) + (B \leftrightarrow C, D)$$

e.g. at four- and five points, have $QC_{1|B,C,D} = 0$ for

$$C_{1|2,3,4} = M_1 M_{2,3,4}$$

$$C_{1|23,4,5} = M_1 M_{23,4,5} + M_{12} M_{3,4,5} - M_{13} M_{2,4,5}$$



BRST covariant building blocks suitable to construct scalar invariants

$$QM_A = \sum_{XY=A} M_X\, M_Y$$

$$QM_{B,C,D} = \sum_{XY=B} (M_X\, M_{Y,C,D} - M_Y\, M_{X,C,D}) + (B \leftrightarrow C, D)$$

e.g. at four- and five points, have $QC_{1|B,C,D} = 0$ for

$$C_{1|2,3,4} = M_1 M_{2,3,4}$$

$$C_{1|23,4,5} = M_1 M_{23,4,5} + \underbrace{M_{12} M_{3,4,5}}_{M_1 \otimes C_{2|3,4,5}} - \underbrace{M_{13} M_{2,4,5}}_{M_1 \otimes C_{3|2,4,5}}$$

$\exists$ recursion for $C_{1|B,C,D} = M_1 M_{B,C,D} +$ BRST-completion using

$$M_A \otimes M_B \equiv M_{AB} \qquad \text{concatenation "inverts" } Q$$

$$C_{1|B,C,D} = M_1 M_{B,C,D} + \Big[M_1 \otimes C_{b_1|b_2 \ldots b_p, C, D} + \text{ 5 others} \Big]$$

Also need vectors to contract loop momentum $\ell_m \leftrightarrow$ zero mode of Π_m:

$$M^m_{A,B,C,D} \;\equiv\; \Big[\underbrace{M_{A,B,C}\mathcal{A}^m_D + (D \leftrightarrow C, B, A)}_{\text{extra zero mode in } \mathcal{U}_D = \Pi_m \mathcal{A}^m_D + \ldots}\Big] + \underbrace{\mathcal{W}^\alpha_A \mathcal{W}^\beta_B \mathcal{W}^\gamma_C \mathcal{W}^\delta_D t^m_{\alpha\beta\gamma\delta}}_{\text{different b-ghost sector}}$$

Also need vectors to contract loop momentum $\ell_m \leftrightarrow$ zero mode of Π_m:

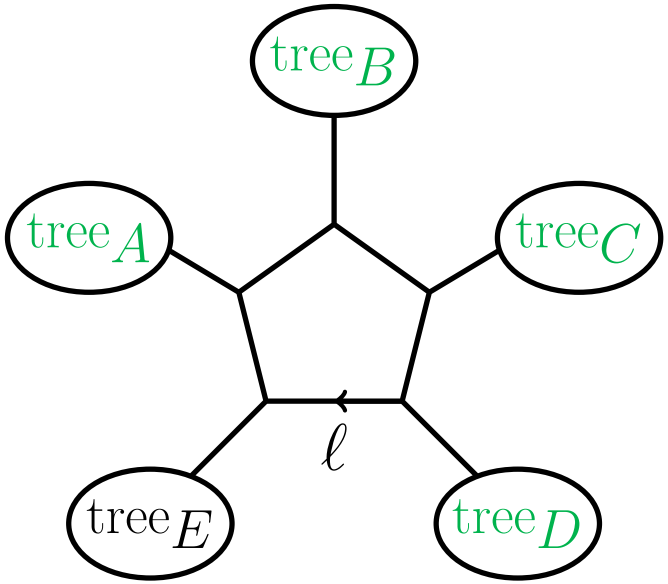
$$M^m_{A,B,C,D} \equiv \left[M_{A,B,C}\mathcal{A}^m_D + (D \leftrightarrow C, B, A) \right] + \mathcal{W}^\alpha_A \mathcal{W}^\beta_B \mathcal{W}^\gamma_C \mathcal{W}^\delta_D \, t^m_{\alpha\beta\gamma\delta}$$

Tensor $t^m_{\alpha\beta\gamma\delta}$ along with $\mathcal{W}^4$ achieves BRST-covariance

$$QM^m_{A,B,C,D} = \sum_{XY=A} \left(M_X \, M^m_{Y,B,C,D} \; - \; M_Y \, M^m_{X,B,C,D} \right)$$

$$+ k^m_A M_A \, M_{B,C,D} \; + \; (A \leftrightarrow B, C, D)$$

$$\langle \, M_E M^m_{A,B,C,D} \, \rangle \quad \leftrightarrow$$



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$$M^m_{A,B,C,D} \equiv \big[M_{A,B,C} \mathcal{A}^m_D + (D \leftrightarrow C, B, A) \big] + \mathcal{W}^\alpha_A \mathcal{W}^\beta_B \mathcal{W}^\gamma_C \mathcal{W}^\delta_D \, t^m_{\alpha\beta\gamma\delta}$$

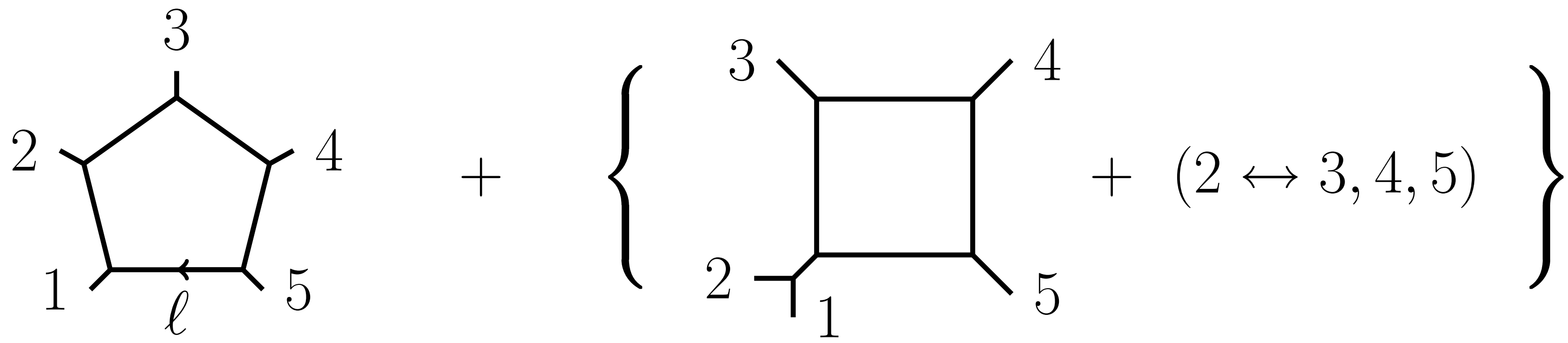
Tensor $t^m_{\alpha\beta\gamma\delta}$ along with $\mathcal{W}^4$ achieves BRST-covariance

$$Q M^m_{A,B,C,D} = \sum_{XY=A} \big(M_X \, M^m_{Y,B,C,D} \; - \; M_Y \, M^m_{X,B,C,D} \big)$$

$$+ k^m_A M_A \, M_{B,C,D} \; + \; (A \leftrightarrow B, C, D)$$

Suitable for vector invariants such as

$$C^m_{1|2,3,4,5} \;=\; M_1 M^m_{2,3,4,5} \; + \; \big[k^m_2 \, M_{12} \, M_{3,4,5} \; + \; (2 \leftrightarrow 3,4,5) \big]$$



Also need vectors to contract loop momentum $\ell_m \leftrightarrow$ zero mode of Π_m:

$$M^m_{A,B,C,D} \;\equiv\; \big[\, M_{A,B,C}\mathcal{A}^m_D + (D \leftrightarrow C, B, A) \,\big] + \mathcal{W}^\alpha_A \mathcal{W}^\beta_B \mathcal{W}^\gamma_C \mathcal{W}^\delta_D \, t^m_{\alpha\beta\gamma\delta}$$

Tensor $t^m_{\alpha\beta\gamma\delta}$ along with $\mathcal{W}^4$ achieves BRST-covariance

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$$+ k^m_A M_A \, M_{B,C,D} \; + \; (A \leftrightarrow B, C, D)$$

Suitable for vector invariants such as

$$C^m_{1|2,3,4,5} \;=\; M_1 M^m_{2,3,4,5} \;+\; \Big[\, \underbrace{k^m_2 \, M_{12} \, M_{3,4,5}}_{k^m_2 M_1 \otimes C_{2|3,4,5}} + (2 \leftrightarrow 3, 4, 5) \,\Big]$$

$\exists$ recursion for $C^m_{1|A,B,C,D}$ in terms of $C_{1|B,C,D}$ and lower point $C^m_{1|\ldots}$

with concatenation $M_A \otimes M_B = M_{AB}$ "inverting" Q.

Higher powers $(\ell)^r$ of loop momentum from r zero modes of $\Pi^m \leftrightarrow \mathcal{A}^m_B$:

$$M^{m_1 m_2 \ldots m_r}_{B_1, B_2, \ldots, B_{r+3}} \equiv M_{B_1, B_2, B_3} (\mathcal{A}^{m_i}_{B_i})^r + \mathcal{W}^4_{B_i} (\mathcal{A}^{m_i}_{B_i})^{r-1}$$

BRST covariant up to anomalous tensor traces:

$$Q M^{m_1 m_2 \ldots m_r}_{B_1, B_2, \ldots, B_{r+3}} = \Big\{ \sum_{XY = B_1} (M_X \, M^{m_1 m_2 \ldots m_r}_{Y, B_2, \ldots, B_{r+3}} - M_Y \, M^{m_1 \ldots m_r}_{X, B_2, \ldots, B_{r+3}})$$

$$+ M_{B_1} k^{(m_1}_{B_1} M^{m_2 \ldots m_r)}_{B_2, \ldots, B_{r+3}} + (B_1 \leftrightarrow B_2, \ldots, B_{r+3}) \Big\} + \delta^{(m_1 m_2} \mathcal{Y}^{m_3 \ldots m_r)}_{B_1, \ldots, B_{r+3}}$$

Higher powers $(\ell)^r$ of loop momentum from r zero modes of $\Pi^m \leftrightarrow \mathcal{A}^m_B$:

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BRST covariant up to anomalous tensor traces:

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$$+ M_{B_1} k^{(m_1}_{B_1} M^{m_2 \ldots m_r)}_{B_2,\ldots,B_{r+3}} + (B_1 \leftrightarrow B_2, \ldots, B_{r+3}) \Big\} + \delta^{(m_1 m_2} \mathcal{Y}^{m_3 \ldots m_r)}_{B_1,\ldots,B_{r+3}}$$

At rank $r = 2$: supersymmetrization of $\varepsilon_{10} F^5$ from hexagon anomaly

[Berkovits, Mafra 0607187]

$$\mathcal{Y}_{A,B,C,D,E} \equiv (\lambda\gamma^m \mathcal{W}_A)(\lambda\gamma^n \mathcal{W}_B)(\lambda\gamma^p \mathcal{W}_C)(\mathcal{W}_D \gamma_{mnp} \mathcal{W}_E)$$

Combine various $M_A M_{B_1,\ldots,B_{j+3}}^{m_1\ldots m_j}$ to tensor superfields such as …

$$C^{mn}_{1|2,3,4,5,6} \;=\; M_1\, M^{mn}_{2,3,4,5,6} \;+\; \big[\, M_{12}\, k_2^{(m}\, M^{n)}_{3,4,5,6} \;+\; (2 \leftrightarrow 3, \ldots, 6)\big]$$
$$-\,\big[\, k_2^{(m} k_3^{n)}\, M_{213}\, M_{4,5,6} \;+\; (23 \leftrightarrow 24, 25, \ldots, 56)\big]$$

… which is BRST invariant up to an anomalous trace

$$Q\, C^{mn}_{1|2,3,4,5,6} \;=\; -\,\delta^{mn}\, M_1\, Y_{2,3,4,5,6}$$ "as invariant as you can get"

Combine various $M_A M^{m_1 \ldots m_j}_{B_1, \ldots, B_{j+3}}$ to tensor superfields such as . . .

$$C^{mn}_{1|2,3,4,5,6} \;=\; M_1\, M^{mn}_{2,3,4,5,6} \;+\; \big[\, M_{12}\, k_2^{(m}\, M^{n)}_{3,4,5,6} \;+\; (2 \leftrightarrow 3, \ldots, 6)\big]$$
$$-\,\big[\, k_2^{(m} k_3^{n)}\, M_{213}\, M_{4,5,6} \;+\; (23 \leftrightarrow 24, 25, \ldots, 56)\big]$$

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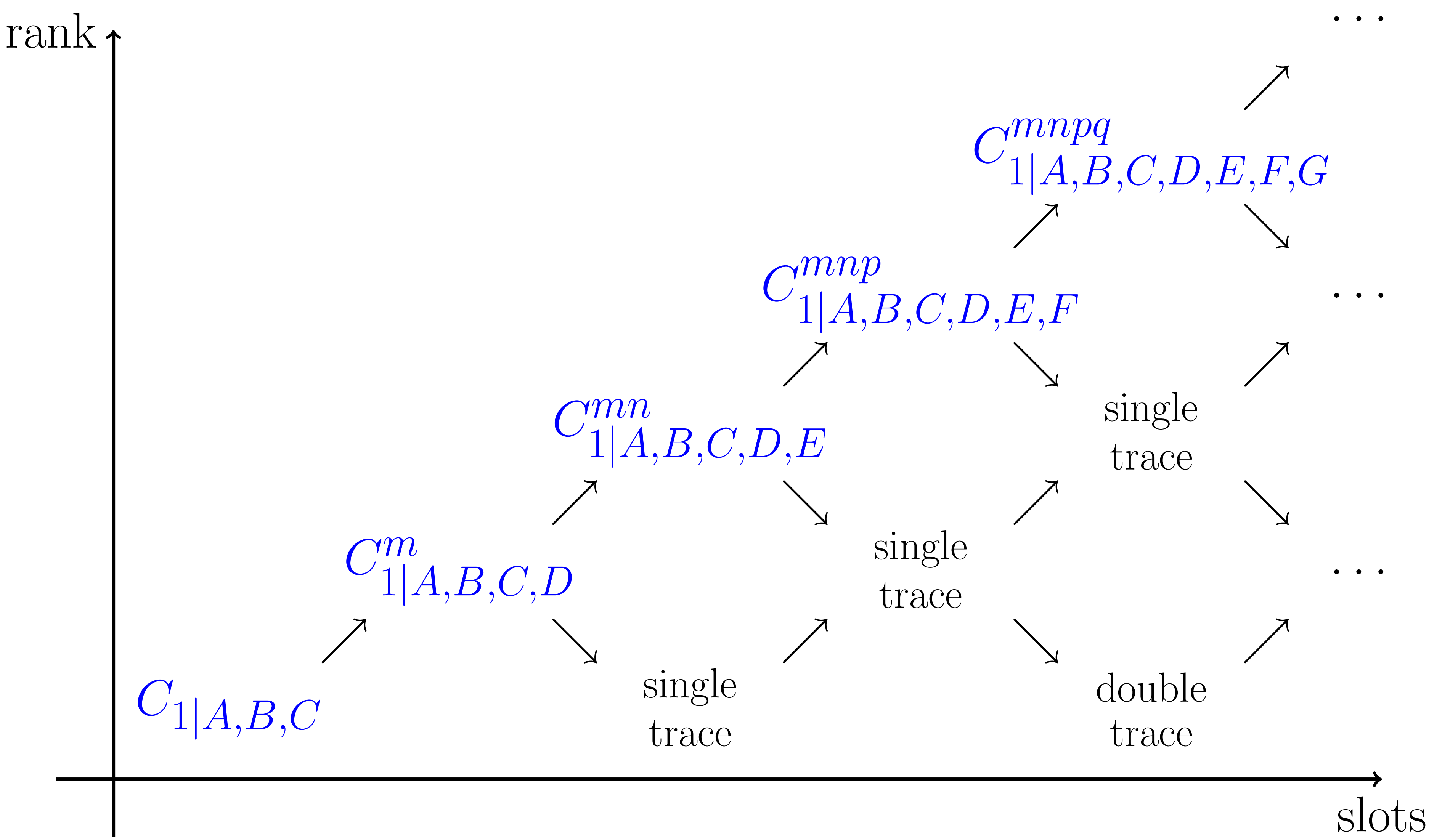
$$Q\, C^{mn}_{1|2,3,4,5,6} \;=\; -\,\delta^{mn}\, M_1\, Y_{2,3,4,5,6} \qquad \text{“as invariant as you can get”}$$

$\Longrightarrow$ “Pseudo–invariants” at any tensor rank

$$C^{m_1 m_2 \ldots m_r}_{1|B_1, B_2, \ldots, B_{r+3}} \equiv\; M_1\, M^{m_1 m_2 \ldots m_r}_{B_1, B_2, \ldots, B_{r+3}} \;+\; \text{recursively found completion}$$
$$Q C^{m_1 m_2 \ldots m_r}_{1|B_1, B_2, \ldots, B_{r+3}} =\; \delta^{m_i m_j} \Big(\text{only “anomalous” terms } M_A\, \mathcal{Y}^{m_k \ldots m_l}_{C_1, C_2, \ldots}\Big)$$



$\Longrightarrow$ "Pseudo–invariants" at any tensor rank **[C. Mafra, OS 1408.3605]**

$$C^{m_1 m_2 \ldots m_r}_{1|B_1,B_2,\ldots,B_{r+3}} \equiv M_1 \, M^{m_1 m_2 \ldots m_r}_{B_1,B_2,\ldots,B_{r+3}} + \text{ recursively found completion}$$

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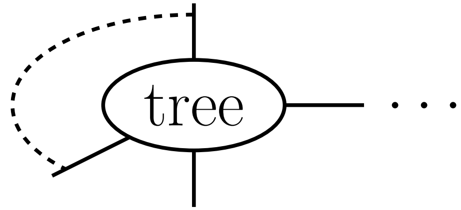
gluon & gluino polariz. e^m, χ^α

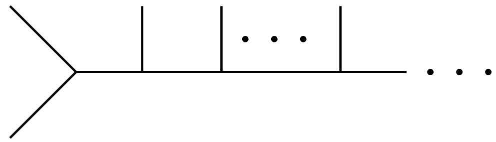
at one–loop: $C_{1|A,B,C}$

and $C^{m_1 \ldots m_r}_{1|B_1,B_2,\ldots,B_{r+3}}$

$M_B,\ \mathcal{A}^m_B,\ \mathcal{W}^\alpha_B,\ \mathcal{F}^{mn}_B$



$A^B_\alpha,\ A^m_B,\ W^\alpha_B,\ F^{mn}_B$



$$A_\alpha(x,\theta) \;=\; e^{ik\cdot x}\Big(\tfrac{1}{2}\, e_m\,(\gamma^m\theta)_\alpha - \tfrac{1}{3}(\chi\gamma_m\theta)(\gamma^m\theta)_\alpha + \theta^3 k e + \theta^4\chi k + \theta^5 k^2 e + \ldots\Big)$$

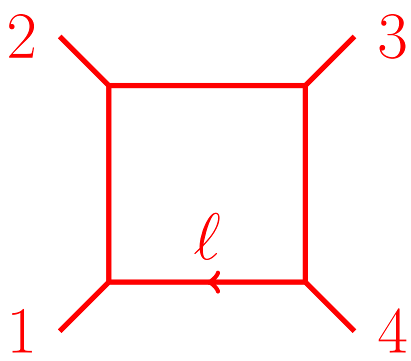
IV. Amplitude examples

Four point one-loop amplitudes in SYM and SUGRA:

$$\mathcal{A}^{1\text{–loop}}(1,2,3,4) = I_{\text{box}}\, C_{1|2,3,4}$$

$$\mathcal{M}_4^{1\text{–loop}} = I_{\text{box}}\, C_{1|2,3,4}\, \tilde{C}_{1|2,3,4}$$

Only the box graph with kinematic factor $\langle C_{1|2,3,4}\rangle = s_{12}s_{23}\mathcal{A}^{\text{tree}}(1,2,3,4)$

$I_{\text{box}} =$ 

$= \int \frac{\mathrm{d}^{10}\ell}{\ell^2(\ell-k_1)^2(\ell-k_{12})^2(\ell-k_{123})^2}$

Originally found from superstring theory.

[Brink, Green, Schwarz 1982]

For five point amplitude: have introduced $6 + 1$ BRST invariants

$$\Big\{ \Big[C_{1|23,4,5} \text{ and } (23 \leftrightarrow 24, 25, 34, 35, 45) \Big] \; ; \quad C^m_{1|2,3,4,5} \Big\}$$

for left- and right movers $\Longrightarrow$ capture polarization dependence of

$$\text{5pt superamplitudes of } \big({\text{gauge} \atop \text{gravity}} \big) \text{ multiplet of } \left({\text{SYM \& SUGRA} \atop \text{superstring}} \right)$$

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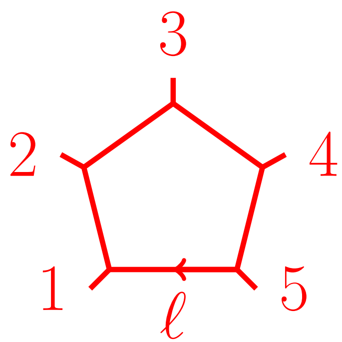
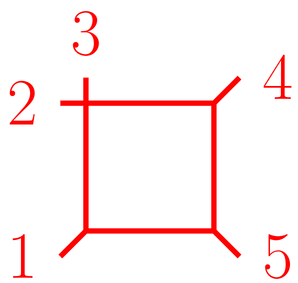
Color ordered SYM amplitude:

$$\mathcal{A}^{1\text{–loop}}(1,2,3,4,5) \;=$$

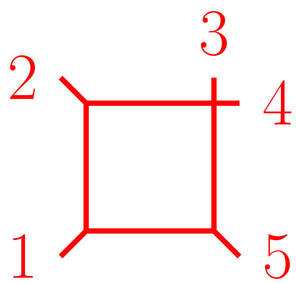


$$\left\{ \begin{array}{c} \ell_m C^m_{1|2,3,4,5} + \frac{1}{2}\big[s_{23} C_{1|23,4,5} \\ + (23 \leftrightarrow 24, 25, 34, 35, 45) \big] \end{array} \right\}$$

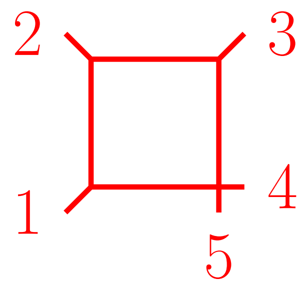
$+$



$C_{1|23,4,5} \; +$

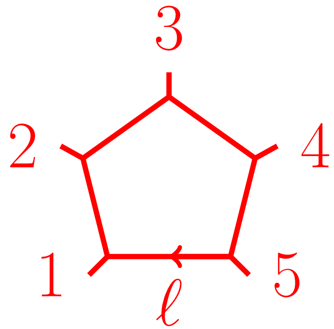


$C_{1|2,34,5} \; +$



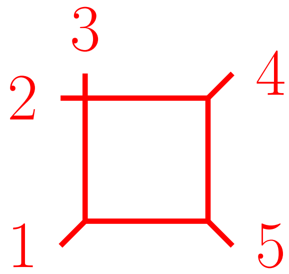
$C_{1|2,3,45}$

$\mathcal{A}^{1\text{–loop}}(1,2,3,4,5) =$

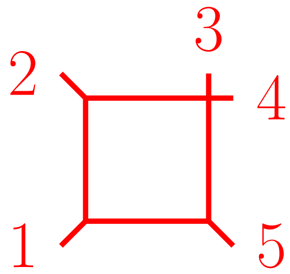


$$\left\{ \ell_m C^m_{1|2,3,4,5} + \tfrac{1}{2}\Big[s_{23} C_{1|23,4,5} + (23 \leftrightarrow 24,25,34,35,45)\Big] \right\}$$

$+$



$C_{1|23,4,5} +$



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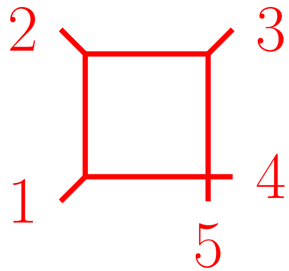
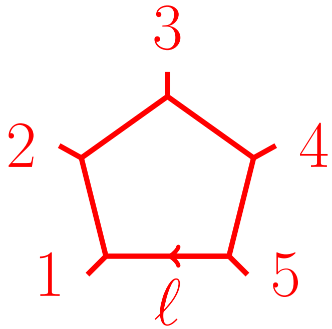


$C_{1|2,3,45}$

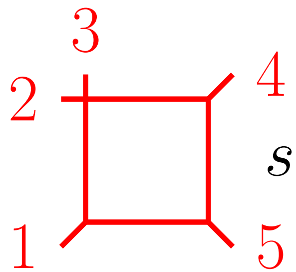
Supergravity amplitude follows by squaring pentagon- & box numerators:

$\mathcal{M}_5^{1\text{–loop}} =$



$$\left\{ \ell_m C^m_{1|2,3,4,5} + \tfrac{1}{2}\Big[s_{23} C_{1|23,4,5} + (23 \leftrightarrow 24,25,34,35,45)\Big] \right\}^2$$

$+$



$s_{23} C_{1|23,4,5} \tilde{C}_{1|23,4,5}$ +permutations in $2,3,4,5$ (not 1)

SYM amplitude descends from string theory ancestor

$$\mathcal{A}^{\text{1-loop}}_{\text{string}}(1,2,3,4,5) \;=\; \int_0^\infty \frac{\mathrm{d}t}{t} \int \mathrm{d}z_2 \ldots \mathrm{d}z_5 \prod_{i<j} e^{\alpha' k_i \cdot k_j G_{ij}}$$

$$\times \; \Big(\frac{\partial}{\partial z_2} G_{23} s_{23}\, C_{1|23,4,5} + (23 \leftrightarrow 24, 25, 34, 35, 45) \Big)$$

with worldsheet Green function $G_{ij} = G(z_i - z_j)$ & cylinder length t.

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$$\times \; \Big(\underbrace{\frac{\partial}{\partial z_2} G_{23} s_{23}\, C_{1|23,4,5} + (23 \leftrightarrow 24, 25, 34, 35, 45)}_{\equiv\, \mathcal{K}^{\text{open}}_5} \Big)$$

with worldsheet Green function $G_{ij} = G(z_i - z_j)$ & cylinder length t.

Closed string is more than naive square of $\mathcal{K}^{\text{open}}_5$ [M. Green, C. Mafra, OS 1307.3534 & in progress]

$$\mathcal{M}^{1\text{–loop}}_{\text{string},5} \;=\; \int_{\mathcal{F}} \frac{\mathrm{d}^2\tau}{\tau_2^5} \int_{\mathcal{T}_\tau} \mathrm{d}^2 z_2 \ldots \mathrm{d}^2 z_5 \prod_{i<j} e^{\alpha' k_i\cdot k_j G_{ij}}$$

$$\times \; \Big(\mathcal{K}^{\text{open}}_5 \, \tilde{\mathcal{K}}^{\text{open}}_5 \;+\; \frac{\pi}{\tau_2} C^m_{1|2,3,4,5}\, \tilde{C}^m_{1|2,3,4,5} \Big)$$

with torus $\mathcal{T}_\tau$, Teichmüller parameter τ and fundamental domain $\mathcal{F}$.

Anomalous six point SYM amplitude

$$\mathcal{A}^{1\text{–loop}}(1,2,\ldots,6) \;=\; \int \frac{\ell_m \ell_n C^{mn}_{1|2,3,4,5,6}\ \mathrm{d}^{10}\ell}{\ell^2\,(\ell-k_1)^2\,(\ell-k_{12})^2\,(\ell-k_{123})^2\,(\ell-k_{1234})^2\,(\ell-k_{12345})^2}$$

$$-\int \frac{P_{1|6|2,3,4,5}\ \mathrm{d}^{10}\ell}{(\ell-k_1)^2\,(\ell-k_{12})^2\,(\ell-k_{123})^2\,(\ell-k_{1234})^2\,(\ell-k_{12345})^2} \;+\; \text{BRST invariant}$$

BRST variations in tensor–hexagon and pentagon $\Rightarrow$ "super $\varepsilon_{10}F^5$"

$$QC^{mn}_{1|2,3,4,5,6} \;=\; -\delta^{mn}V_1 Y_{2,3,4,5,6}\;, \qquad QP_{1|6|2,3,4,5} = -V_1 Y_{2,3,4,5,6}$$

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$$Q\mathcal{A}^{1\text{–loop}}(1,2,\ldots,6) \;=\; V_1 Y_{2,3,4,5,6} \int \mathrm{d}^{10}\ell \left(1 \;-\; \frac{\delta_{mn}\ell^m \ell^n}{\ell^2}\right)$$

$$\times\ \frac{1}{(\ell-k_1)^2\,(\ell-k_{12})^2\,(\ell-k_{123})^2\,(\ell-k_{1234})^2\,(\ell-k_{12345})^2}$$

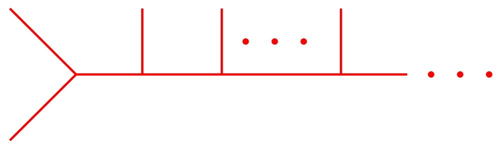
Even though the integrand formally vanishes, get finite and rational anomaly

$$Q\mathcal{A}^{1\text{–loop}}(1,2,\ldots,6) \;=\; -\frac{\pi^5}{5!}\, V_1 Y_{2,3,4,5,6}$$

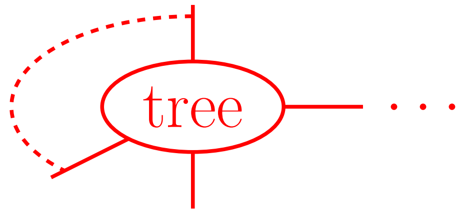
V. Conclusion & Outlook

- Identified LEGO–brickstones of tree subamplitudes

$$A^B_\alpha\,,\ A^m_B,\ W^\alpha_B,\ F^{mn}_B$$



$$M_B,\ \mathcal{A}^m_B,\ \mathcal{W}^\alpha_B,\ \mathcal{F}^{mn}_B$$



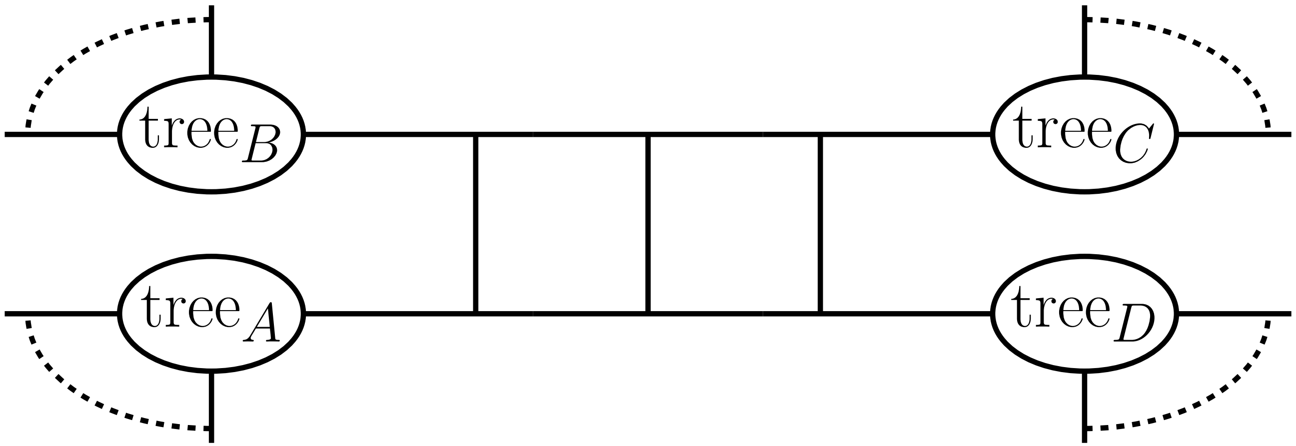
- found systematic & unique BRST–completion @ tree level and 1–loop

$$C^{m_1\ldots m_r}_{1|B_1,B_2,\ldots,B_{r+3}} \quad = \quad M_1\, M^{m_1\ldots m_r}_{B_1,B_2,\ldots,B_{r+3}} \ + \ \text{recursive completion}$$

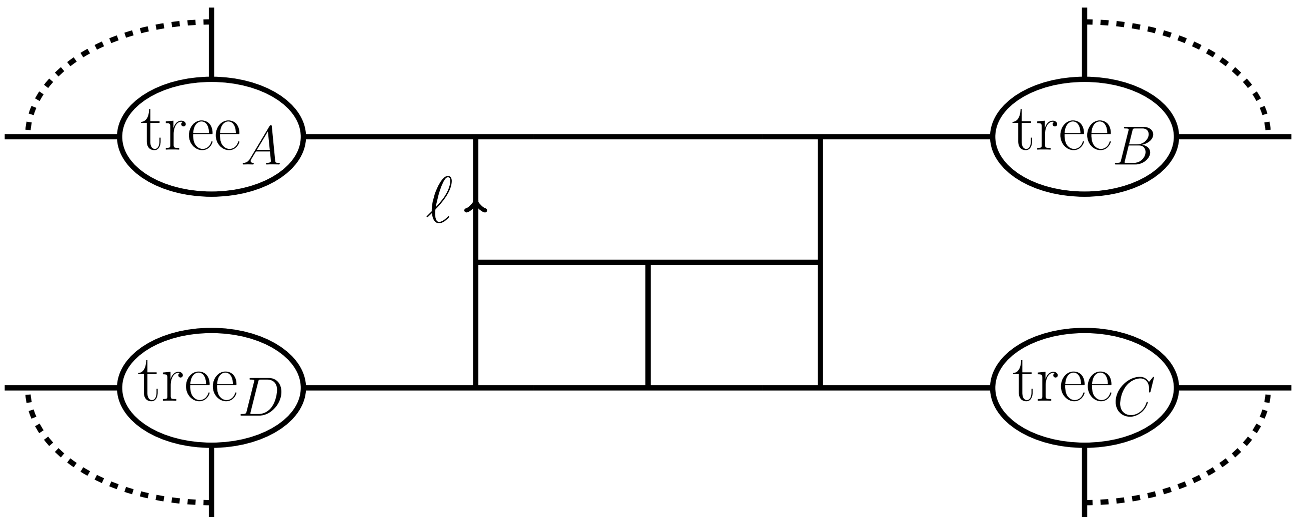
- BRST non–invariant traces reflect hexagon anomaly

$$QC^{m_1\ldots m_r}_{1|B_1,B_2,\ldots,B_{r+3}} \quad = \quad -M_1\, \delta^{(m_1 m_2}\, \mathcal{Y}^{m_3\ldots m_r)}_{B_1,\ldots,B_{r+3}} \ + \ \text{recursive completion}$$

Work in progress: extend superfield mappings to higher loops, e.g.

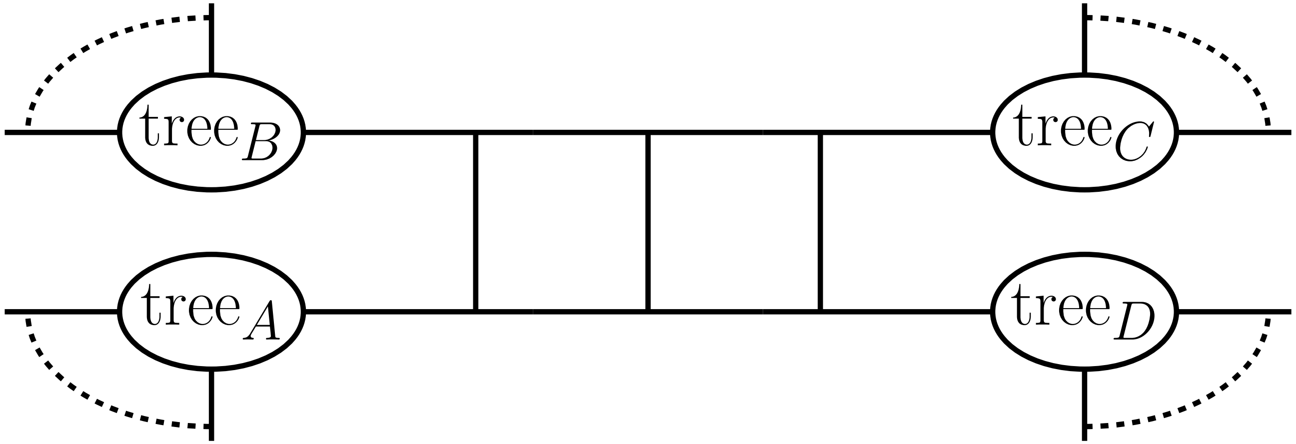


$$\longleftrightarrow \quad M_{A,B|C,D}$$



$$\longleftrightarrow \quad M_{[A,B]|C|D} + \ell_m \, M^m_{A,B|C|D}$$

Work in progress: extend superfield mappings to higher loops, e.g.



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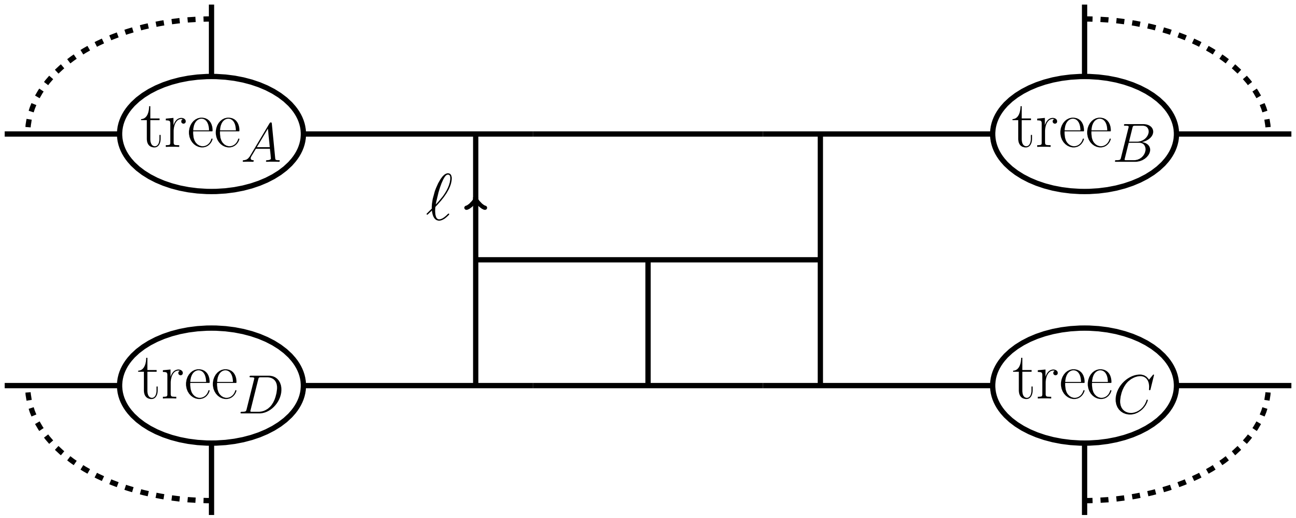

$$\longleftrightarrow \quad M_{[A,B]|C|D} + \ell_m \, M^m_{A,B|C|D}$$

Thank you for your attention !