Correlation functions of stress-tensor multiplets in N=4 SYM

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New Geometric Structures in Scattering Amplitudes
September 25th, 2014
Oxford

based on work with:
Eden, Korchemsky, Sokatchev (arxiv:1108.3557,1201.5329)
Ambrosio, Eden, Goddard, Taylor (arXiv:1312.1163)
Chicherin, Doobary, Eden, Korchemsky, Mason, Sokatchev, to appear soon

Outline

Four-point correlation functions in planar $\mathcal{N} = 4$ SYM

- Summarise progress over last three years

Amplitudes in planar $\mathcal{N} = 4$ SYM

- four- and five- point amplitude integrand from 4 point correlator

Higher point correlators: Twistor approach

Correlators

(Correlation functions of gauge invariant operators)

- *Gauge invariant operators:* gauge invariant products (ie traces) of the fundamental fields
- Simplest operator trϕ^2 (ϕ one of the scalars)
- The simplest *non-trivial* correlation function is

$$G_4 := \langle \mathcal{O}(x_1)\bar{\mathcal{O}}(x_2)\mathcal{O}(x_3)\bar{\mathcal{O}}(x_4)\rangle \qquad \mathcal{O} = \mathrm{Tr}(\phi_{12}\phi_{12})$$

- $\mathcal{O} \in$ stress energy supermultiplet. We consider correlators of all operators in this multiplet.
- Later discuss higher points too

Why are they interesting?

AdS/CFT

Supergravity/String theory on $AdS_5 \times S^5 \quad = \quad \mathcal{N}$=4 super Yang-Mills

- Correlation functions of gauge invariant operators in SYM $\leftrightarrow$ string scattering in *AdS*
- Contain data about anomalous dimensions of operators and 3 point functions via OPE $\rightarrow$integrability / bootstrap
- Finite
- Big Bonus of last 3 years Correlators contain all scattering amplitudes (more later)

Analytic superspace

- Stress-tensor multiplet best described using analytic superspace [GIKOS, Hartwell Howe]

Analytic superspace = Grassmannian of (2|2) planes in $C^{4|4}$

$$\left(\begin{array}{c|c} z_i & \theta_i \\ \hline 0 & u_i \end{array}\right) \sim \left(\begin{array}{c|c} a_i & \beta_i \\ \hline 0 & c_i \end{array}\right)\left(\begin{array}{c|c} z_i & \theta_i \\ \hline 0 & u_i \end{array}\right)$$

- z: 2×4 matrix, Grassmannian of 2 planes in C^4 = Minkowski space = lines in twistor space
- u: 2×4 matrix, Grassmannian of 2 planes in C^4 = internal space
- Solve odd part of local $Sl(2|2)$: $\theta_i \sim \theta_i + \beta_i u_i \quad \Rightarrow \quad \rho_i := \theta_i \bar{u}_i \sim \rho_i$
- 6d coordinates for Minkowski and internal space

$$(X_i)_{AB} = (z_i)^\alpha{}_A (z_i)^\beta{}_B \epsilon_{\alpha\beta} \qquad (\bar{X}_i)^{AB} = (\bar{z}_i)^A{}_{\dot\alpha} (\bar{z}_i)^B{}_{\dot\beta} \epsilon^{\dot\alpha\dot\beta} = \frac{1}{2}\epsilon^{ABCD}(X_i)_{AB}$$

$$(Y_i)_{IJ} = (u_i)^a{}_I (u_i)^b{}_J \epsilon_{ab} \qquad (\bar{Y}_i)^{IJ} = (\bar{u}_i)^I{}_a (\bar{u}_i)^J{}_b \epsilon^{ab} = \frac{1}{2}\epsilon^{IJKL}(Y_i)_{KL}\,.$$

In the standard chart corresponding to the usual space-time x coordinates and corresponding internal coordinates y we put:

Standard chart

$$z_i = (1_2 \ x_i) \quad \hat{z}_i = (0_2 \ 1_2) \qquad \Rightarrow \quad \hat{\bar{z}}_i = \begin{pmatrix} 1_2 \\ 0_2 \end{pmatrix} \quad \bar{z}_i = \begin{pmatrix} -x_i \\ 1_2 \end{pmatrix}$$

$$u_i = (1_2 \ y_i) \quad \hat{u}_i = (0_2 \ 1_2) \qquad \Rightarrow \quad \hat{\bar{u}}_i = \begin{pmatrix} 1_2 \\ 0_2 \end{pmatrix} \quad \bar{u}_i = \begin{pmatrix} -y_i \\ 1_2 \end{pmatrix}$$

$$\theta_i = (0_2 \ \rho_i) \; .$$

Superspace: correlation functions

stress-energy supermultiplet (half BPS)

$$\mathcal{T}(x,\rho,\bar\rho=0,y) = \mathcal{O}(x,y) + \ldots + \rho^4 \mathcal{L}(x),$$
$$\mathcal{O}(x,y) = Tr(\phi_{IJ}\phi_{KL}) Y^{IJ} Y^{KL}$$

- correlation function of $\mathcal{T}$s: ρ-expansion organised in powers of ρ^{4k}

Superspace expansion (cf superamplitude)

$$G_n|_{\bar\rho=0} := \langle \mathcal{T}(1)\mathcal{T}(2)\ldots\mathcal{T}(n)\rangle$$
$$= G_{n;0} + \left[\rho^4\right] G_{n;1} + \left[\rho^8\right] G_{n;2} + \cdots + \left[\rho^{4(n-4)}\right] G_{n;n-4}$$

Integrands = correlators with Lagrangian insertions

- Loop corrections $\Rightarrow$ Lagrangian insertions.

1 loop correlator

$$\langle \mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(1)} = \int d^4x_0\,\langle \mathcal{L}(x_0)\mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(0)}$$
$$= \int d^4x_0 d^4\rho_0 \langle \mathcal{T}(0)\mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(0)}$$

- so the *Born-level* $(n+1)$-point correlator defines the 1 loop integrand: $G^{(1)}_{n;k} = G^{(0)}_{n+1;k+1}$
- ℓ-loops $\Rightarrow$ ℓ Lagrangian insertions $\Rightarrow$ $n+\ell$-point tree correlator

$$G^{(\ell)}_{n;k} = G^{(\ell-1)}_{n+1;k+1} = \dots = G^{(0)}_{n+\ell;k+\ell}$$

- NB parameter space $n,k,\ell \to n,k$
- Amplitudes need n,k,ℓ: Many different amplitudes contained in each correlator.

Hidden permutation symmetry for the four-point correlator

- $G^{(0)}_{n;n-4}$ is "tree-level $\overline{\text{MHV}}$" correlator
- $G_{n;n-4} =$(S_n symmetric $\rho^{4(n-4)}$ invariant) $\times f(x_i)$
- Crossing symmetry of super correlator under simultaneous $(x_i, \rho_i, y_i) \to (x_j, \rho_j, y_j) \Rightarrow$ permutation symmetry of $f(x_i)$

But $G^{(0)}_{n;n-4}$ is the four-point $(n-4)$-loop integrand

Four-point correlator is given in terms of

$$f(x_i; a) = \sum_{\ell=1}^{\infty} \frac{a^\ell}{\ell!} \int d^4x_5 \dots d^4x_{4+\ell}\, f^{(\ell)}(x_1, \dots, x_{4+\ell})$$

Hidden symmetry:

$$f^{(\ell)}(x_1, \dots x_{4+\ell}) = f^{(\ell)}(x_{\sigma_1}, \dots x_{\sigma_{4+\ell}}) \qquad \forall\, \sigma \in S_{4+\ell}$$

- The symmetry mixes external variables $x_1, \dots x_4$ with integration variables $x_5 \dots x_{4+\ell}$

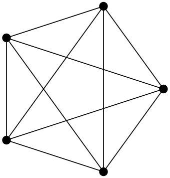
1-, 2- and 3-loop integrands

- Entire 4-pnt correlator defined (perturbatively) via $f^{(\ell)}$
  - conformal weight 4 at each point
  - permutation invariant
  - No double poles (from OPE)
- Naively equivalent to: degree (valency) 4 graphs on $4+\ell$ points

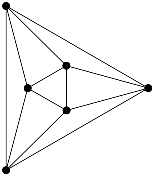
$$\text{graph edge} = \frac{1}{x_{ij}^2}$$

- (But: we are also allowed numerator lines $\Rightarrow$ degree $\geq$ 4 graphs).
- Don't need to label graph since we sum over permutations $\Rightarrow$ sum over all possible ways of labelling

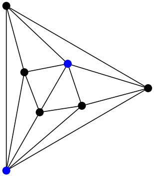
$$f^{(1)} = \frac{1}{\prod_{1\leq i<j\leq 5} x_{ij}^2}$$



$$f^{(2)} = \frac{x_{12}^2 x_{34}^2 x_{56}^2 \ + \ S_6 \text{ perms}}{\prod_{1\leq i<j\leq 6} x_{ij}^2}$$

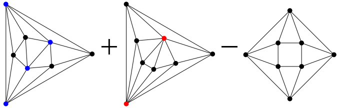


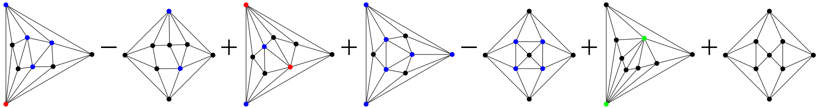
$$f^{(3)} = \frac{(x_{12}^4)(x_{34}^2 x_{45}^2 x_{56}^2 x_{67}^2 x_{73}^2) \ + \ S_7 \text{ perms}}{\prod_{1\leq i<j\leq 7} x_{ij}^2}$$



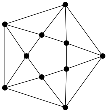
Unique (planar) possibilities

Four- and five-loops

$f^{(4)} =$ 

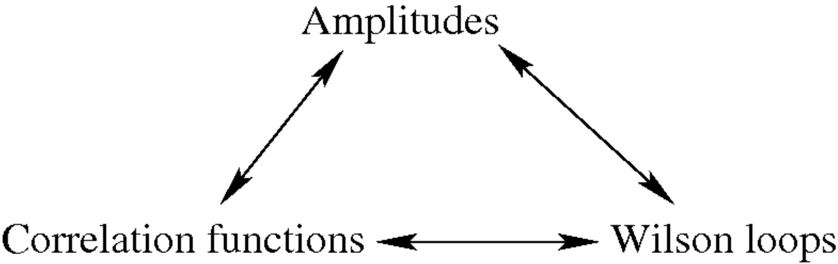
$f^{(5)} =$ 

- Very compact writing
- All come with coefficients 1,-1 (determined using technique of [Bourjaily, DiRe, Shaikh, Spradlin, Volovich])
- From 6 loops we start to see integrands with the coefficient 2 (and also 0), the first being:



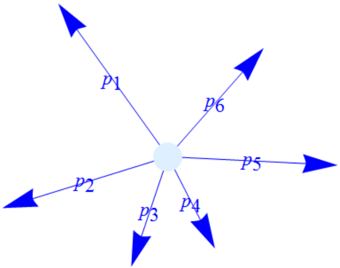
Relation to amplitudes

- triality between three objects in $\mathcal{N} = 4$ SYM

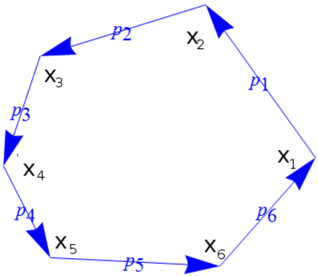


[Alday Maldacena, Drummond Henn Korchemsky Sokatchev, Brandhuber Travaglini PH, Mason Skinner, Caron-Huot, Alday Eden Korchemsky Maldacena Sokatchev, Eden Korchemsky Sokatchev PH, Adamo Bullimore Mason Skinner, ...]

Triality



$$\frac{\text{Full planar superamplitude}}{\text{MHV tree}}$$



super Wilson loop (vev)

$$\lim_{x_{i\,i+1}^2 \to 0} \frac{\langle \mathcal{T}(x_1,\rho_1,y_1)\mathcal{T}(x_2,\rho_2,y_2)\ldots\mathcal{T}(x_n,\rho_n,y_n)\rangle}{\langle \mathcal{T}(x_1,\rho_1,y_1)\mathcal{T}(x_2,\rho_2,y_2)\ldots\mathcal{T}(x_n,\rho_n,y_n)\rangle_{\text{tree}}}$$

Full super-correlation function (ys completely factorise)

Superspaces: superamplitudes

- Use Nair's $\mathcal{N}$=4 on-shell superspace, all particles $\rightarrow$ superparticle

super-particle

$$\Phi(p,\eta) = G^+(p) + \eta\psi + \eta^2\phi(p) + \eta^3\bar{\psi}(p) + \eta^4 G^-(p)$$

- All amplitudes $\rightarrow$ superamplitudes

$$A(x_i) \rightarrow \mathcal{A}(x_i, \eta_i)$$

super-amplitude structure: $\mathcal{A}(x_i, \eta_i) =$

$$[\eta^8] A_{MHV} + [\eta^{12}] A_{NMHV} + [\eta^{16}] A_{NNMHV} + \ldots + [\eta^{4(n-2)}] A_{\overline{MHV}}$$
$$= A^{\text{tree}}_{MHV} \left(\hat{A}_{MHV} + [\eta^4] \hat{A}_{NMHV} + [\eta^8] \hat{A}_{NNMHV} + \ldots + [\eta^{4(n-4)}] \hat{A}_{\overline{MHV}} \right)$$

Superamplitude/ supercorrelation function duality

Superduality

$$\lim_{x^2_{i\,i+1}\to 0} \frac{\langle \mathcal{T}(1)\ldots\mathcal{T}(n)\rangle}{\langle \mathcal{T}(1)\ldots\mathcal{T}(n)\rangle^{\text{tree}}_{n;0}}(x,\rho,\bar\rho=0,y) \;=\; \left(\frac{\mathcal{A}_n}{\mathcal{A}^{\text{tree}}_{n;\text{MHV}}}(x,\eta)\right)^2$$

- duality works at the level of the integrand...
- Amplitude written in terms of dual/region momenta
- $p_i = x_i - x_{i+1}$
- OR better momentum twistors (Hodges). Points where consecutive supertwistor lines intersect in the lightlike limit.

Correlator amplitude duality at 4,5 points

- But $G^{(\ell)}_{5;1} = G^{(\ell+1)}_{4;0}$ at the integrand level
- Both four-point and five-point amplitudes are given in terms of the same objects: f-graphs

- external factor $\times \lim_{\substack{x^2_{i\,i+1}\to 0 \\ (\text{mod } 4)}} \int d^4x_5 \dots d^4x_{4+\ell} \frac{f^{(\ell)}}{\ell!} := F^{(\ell)}_4 = (\mathcal{M}_4)^2$
- external factor $\times \lim_{\substack{x^2_{i\,i+1}\to 0 \\ (\text{mod } 5)}} \int d^4x_6 \dots d^4x_{5+\ell} \frac{f^{(\ell+1)}}{\ell!} := F^{(\ell)}_5 = 2\mathcal{M}_5\overline{\mathcal{M}}_5$

- We can determine four points and five-point amplitude completely from the four-point correlator
- Beyond 5 points eg 6-point limit = $\mathcal{M}_6\overline{\mathcal{M}}_6 + NMHV^2$.

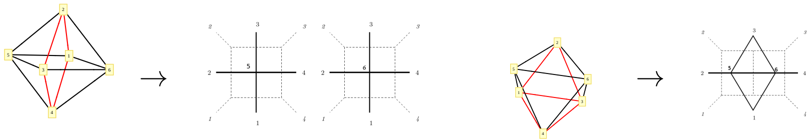
Amplitude information from $f^{(2)}$ (octahedron)

- Having expanded in fermionic variables, we now expand in the coupling a: $\mathcal{M}_n := 1 + a\mathcal{M}_n^{(1)} + a^2\mathcal{M}_n^{(2)} + a^3\mathcal{M}_n^{(3)} + \ldots$

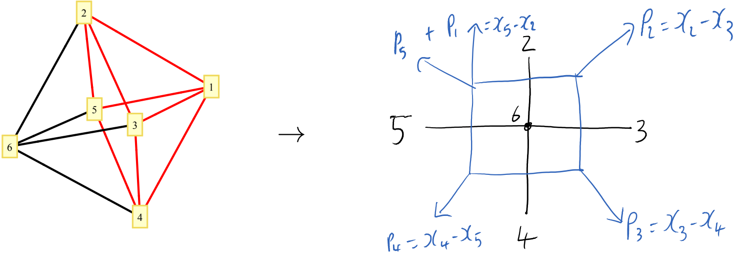
Octahedron $f^{(2)} \;\rightarrow\; F_4^{(2)}, F_5^{(1)}$

$$F_4^{(2)} = 2\mathcal{M}_4^{(2)} + \left(\mathcal{M}_4^{(1)}\right)^2 \qquad\qquad F_5^{(1)} = \mathcal{M}_5^{(1)} + \overline{\mathcal{M}}_5^{(1)}$$

Graphically at four points:



Graphically at **five**-points (one loop):

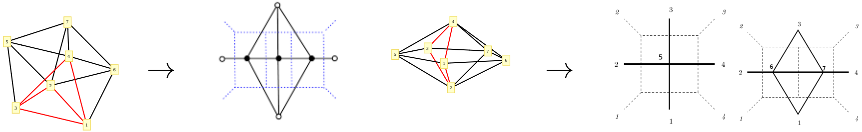


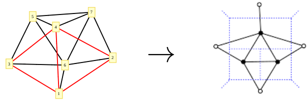
- Summing all permutations gives the sum over 1 mass box functions = parity even 1-loop 5-point amplitude

- Also a well known parity odd part $O(\epsilon)$ but important eg in BDS
- To see this let's consider the next order...

Four-points, 3 loop (from $f^{(3)}$)

We have $F_4^{(3)} = 2\mathcal{M}_4^{(3)} + \mathcal{M}_4^{(1)}\mathcal{M}_4^{(2)}$

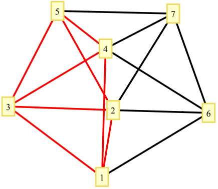




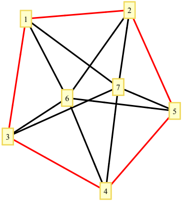
- Graphically: the four external points we pick need to be connected consecutively: four-cycle.
- Four-cycle splits the planar graph into two pieces. Correspond to product terms.

Five-points, 2 loop and parity odd 1 loop (from $f^{(3)}$)

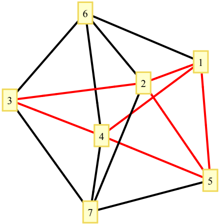
- We have $F_5^{(2)} = \mathcal{M}_5^{(2)} + \overline{\mathcal{M}}_5^{(2)} + \mathcal{M}_5^{(1)}\overline{\mathcal{M}}_5^{(1)}$
- Distinguish contributions by topology: If the 5-cycle “splits” the f-graph it contributes to $\mathcal{M}_5^{(1)}\overline{\mathcal{M}}_5^{(1)}$ otherwise to $\mathcal{M}_5^{(2)} + \overline{\mathcal{M}}_5^{(2)}$



2 loop ladder



pentagon^2



box$\times$ box

- Two equations ($\mathcal{M}_5^{(1)} + \overline{\mathcal{M}}_5^{(1)} = \sum$ boxes; $\mathcal{M}_5^{(1)}\overline{\mathcal{M}}_5^{(1)} =$ products).
- Solve eqns gives the full (parity even and odd) amplitude $\mathcal{M}_5^{(1)}$.

The equation is quadratic and has solution

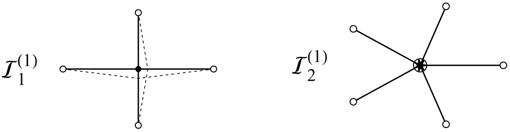
$$M_5^{(1)} = \frac{1}{2}\left(F_5^{(1)} \pm \sqrt{(F_5^{(1)})^2 - 4F_{5,\text{products}}^{(2)}}\right)$$

- One can check that this simplifies very nicely to:

$$M_5^{(1)} = \frac{1}{2}\left(\mathcal{I}_1^{(1)} + \mathcal{I}_2^{(1)}\right) \ .$$

$$\mathcal{I}_1^{(1)} = \text{cyc}\left[\frac{x_{13}^2 x_{25}^2}{x_{16}^2 x_{26}^2 x_{36}^2 x_{56}^2}\right] \quad \mathcal{I}_2^{(1)} = \text{cyc}\left[\frac{i\epsilon_{123456}}{x_{16}^2 x_{26}^2 x_{36}^2 x_{46}^2 x_{56}^2}\right]$$

- The terms are displayed graphically as



- The starred vertex v indicates a factor $i\epsilon_{12345v}$.

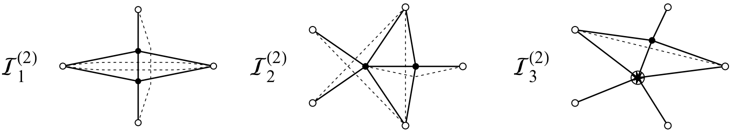
Story continues to higher loops (from $f^{(3)}$ and $f^{(4)}$)

$$F^{(\ell)} = M_5^{(\ell)} + \overline{M}_5^{(\ell)} + \sum_{m=1}^{\ell-1} M_5^{(m)} \overline{M}_5^{(\ell-m)}$$

$$F^{(\ell+1)} = M_5^{(\ell+1)} + \overline{M}_5^{(\ell+1)} + M_5^{(\ell)} \overline{M}_5^{(1)} + M_5^{(1)} \overline{M}_5^{(\ell)} + \sum_{m=2}^{\ell-1} M_5^{(m)} \overline{M}_5^{(\ell-m+1)}$$

The full two-loop amplitude is

$$M_5^{(2)} = \frac{1}{2 \times 2!} \left(\mathcal{I}_1^{(2)} + \mathcal{I}_2^{(2)} + \mathcal{I}_3^{(2)} \right)$$



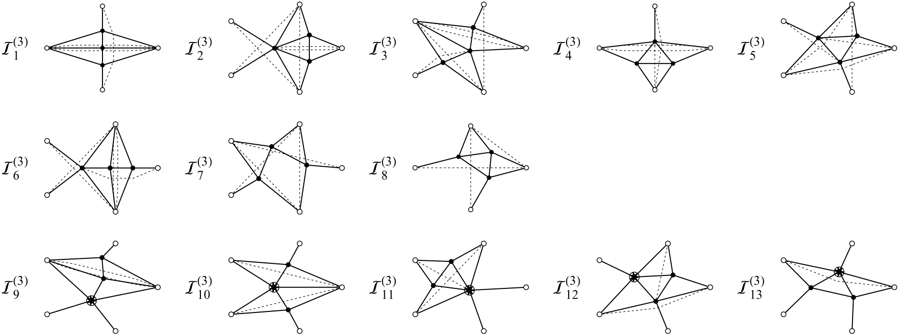
(To help find the result we have conjectured that the only parity odd object is ϵ_{12345v} (Never get two internal variables in an ϵ.)

...and higher loops (from $f^{(4)}$ and $f^{(5)}$)

The full three-loop amplitude is

$$M_5^{(3)} = \frac{1}{2.3!}\int d^4x_6 d^4x_7 d^4x_8 \left(\sum_{i=1}^{13} c_i \mathcal{I}_i^{(3)}\right)$$

$$c_1 = \cdots = c_6 = c_9 = \ldots c_{12} = 1 \qquad c_7 = c_8 = c_{13} = -1$$



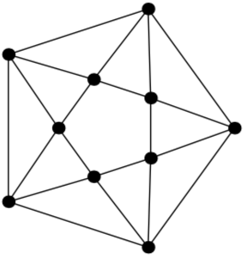
...and higher loops

- We have up to $f^{(7)}$ and thus we have $\mathcal{M}_5^{(5)}$ completely and $\mathcal{M}_5^{(6)}$ (parity even part).

- Understand how cancellation of non-planar graphs works

- Construction determines correlator coefficients (extension of rung rule - just consistency determines everything up to $f^{(5)}$)

(NB But still not the intriguing $f^{(6)}$ graph occurring with coefficient 2)



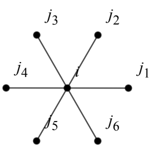
Higher point correlation functions?

- So far: four-point high loop correlators ($(4+\ell)$-point correlators at $\rho^{4\ell}$, max nilpotent)
- Can we go to higher points ie non-maximally nilpotent?
- Yes ... using twistor space

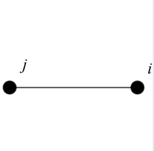
Recall Twistor Wilson loop

[Bullimore Mason Skinner]

Twistor Wilson loop Feynman rules



$$\rightarrow \quad \int d^4 x_i d^8 \theta_i \, \frac{1}{\left(\prod_k \sigma_{ij_k} \cdot \sigma_{ij_{k+1}}\right)} \qquad \text{(internal vertex)}$$



$$\rightarrow \quad \int d^2 \sigma_{ij} d^2 \sigma_{ji} \, \delta^{4|4}\left(\mathcal{Z}^* + \sigma_{ij\alpha} \mathcal{Z}_i^{\alpha} + \sigma_{ji\alpha} \mathcal{Z}_j^{\alpha}\right)$$

- For Wilson loop modified contribution when propagator ends on the WL – “external vertex” .
- Here I represent twistor lines $\mathcal{Z}_i^{\alpha}$ graphically as points (space-time picture)

Key observation for stress-energy correlator

(See [Adamo Bullimore Mason Skinner] for other operators)

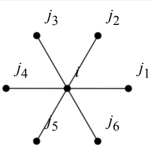
- Internal vertex corresponds to insertion of the action as usual
- Action = $g^2 \int d^4x d^8\theta \log\det \mathcal{D}|_x$ on twistor space
- But Action = $g^2 \int d^4x d^4\rho\, \mathcal{T}(x, \rho, \bar{\rho}, Y)$
- Therefore $\mathcal{T} = \int d^4\hat{\rho} \log\det \mathcal{D}|_x$ (where $d^8\theta = d^4\hat{\rho}\, d^4\rho$)

Stress-energy correlator:

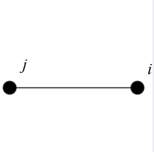
$$\langle \mathcal{T}_1 \dots \mathcal{T}_n \rangle = \left(\prod_{i=1}^{n} \int d^4\hat{\rho} \right) \times \sum n\text{-pt vacuum diagrams}$$

(ie no external Wilson loop) with $\int d^4x_i d^8\theta_i \to \int d^4\hat{\rho}_i$

Wilson loop Feynman rules

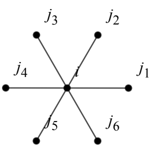


$$\rightarrow \quad \int d^4 x_i d^8 \theta_i \, \frac{1}{(\prod_k \sigma_{ij_k} \cdot \sigma_{ij_{k+1}})}$$

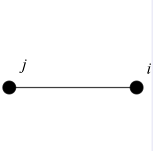


$$\rightarrow \quad \int d^2 \sigma_{ij} d^2 \sigma_{ji} \; \delta^{4|4}(\mathcal{Z}^* + \sigma_{ij\alpha} \mathcal{Z}_i^{\alpha} + \sigma_{ji\alpha} \mathcal{Z}_j^{\alpha})$$

Correlator Feynman rules



$$\rightarrow \quad \int d^4\hat{\rho}_i \qquad \frac{1}{\left(\prod_k \sigma_{ij_k}\cdot\sigma_{ij_{k+1}}\right)}$$



$$\rightarrow \quad \int d^2\sigma_{ij} d^2\sigma_{ji}\ \delta^{4|4}(\mathcal{Z}^* + \sigma_{ij\alpha}\mathcal{Z}_i^\alpha + \sigma_{ji\alpha}\mathcal{Z}_j^\alpha)$$

Superspace Feynman rules on analytic superspace

Systematically perform the fermionic integration $\int d^4\hat{\rho}_i$

1. Split the fermionic delta function into two δ^2 bits by projecting with internal coordinates to expose $\hat{\rho}$

$$\delta^4(\sigma_{ij}\theta_i + \sigma_{ji}\theta_j + \theta_*) = (Y_i.Y_j)$$
$$\times\, \delta^2\Big(\sigma_{ji}\hat{\rho}_j + \overbrace{\sigma_{ij}\rho_i(u_j\bar{u}_i)^{-1} + \sigma_{ji}\rho_j\hat{u}_j\bar{u}_i(u_j\bar{u}_i)^{-1} + \theta_*\bar{u}_i(u_j\bar{u}_i)^{-1}}^{A_{ij}}\Big)$$
$$\times\, \delta^2\Big(\sigma_{ij}\hat{\rho}_i + \sigma_{ji}\rho_j(u_i\bar{u}_j)^{-1} + \sigma_{ij}\rho_i\hat{u}_i\bar{u}_j(u_i\bar{u}_j)^{-1} + \theta_*\bar{u}_j(u_i\bar{u}_j)^{-1}\Big)$$

2. Apply $\int d^4\hat{\rho}_i$
3. Do the parameter (σ) integrals:

$$\int d^2\sigma_{ij}d^2\sigma_{ji}\,\delta^{4|0}(Z^* + \sigma_{ij}Z_i + \sigma_{ji}Z_j) \;=\; \frac{1}{X_i\cdot X_j}$$

$$\text{freezes} \qquad \sigma_{ij\alpha} = \frac{\langle *z_{i\alpha}X_j\rangle}{X_i\cdot X_j}$$

Feynman rules

Easy case, 2-valent vertex: vertex i attached to vertices j and k only:

$\int d^4\hat{\rho}_i \delta^2(\sigma_{ij}\hat{\rho}_i + A_{ij})\delta^2(\sigma_{ik}\hat{\rho}_i + A_{ik}) = \langle\sigma_{ij}\sigma_{ik}\rangle^2$, Precisely cancels the denominator factor from the Feynman rules.

Hard case: 3-valent vertex $R(i; j_1 j_2 j_3; *) :=$

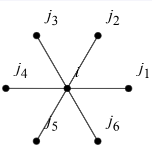
$$\int d^4\hat{\rho}_i \frac{\delta^2(\sigma_{ij_1}\hat{\rho}_i + A_{ij_1})\delta^2(\sigma_{ij_2}\hat{\rho}_i + A_{ij_2})\delta^2(\sigma_{ij_3}\hat{\rho}_i + A_{ij_3})}{\langle\sigma_{ij_1}\sigma_{ij_2}\rangle\,\langle\sigma_{ij_2}\sigma_{ij_3}\rangle\,\langle\sigma_{ij_3}\sigma_{ij_1}\rangle}$$

$$= \frac{\delta^2\Big(\langle\sigma_{ij_1}\sigma_{ij_2}\rangle A_{ij_3} + \langle\sigma_{ij_2}\sigma_{ij_3}\rangle A_{ij_1} + \langle\sigma_{ij_3}\sigma_{ij_1}\rangle A_{ij_2}\Big)}{\langle\sigma_{ij_1}\sigma_{ij_2}\rangle\langle\sigma_{ij_2}\sigma_{ij_3}\rangle\,\langle\sigma_{ij_3}\sigma_{ij_1}\rangle}$$

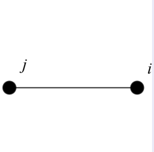
- Higher valency vertices are products of the 3-valent case

Feynman rules on analytic superspace:

Analytic superspace Feynman rules



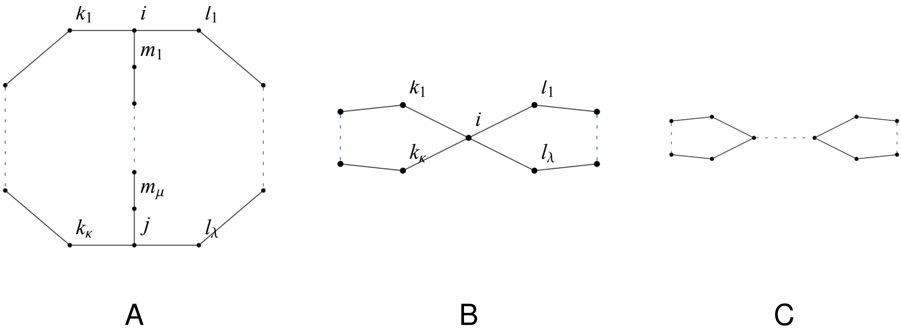
$$\rightarrow \quad R(i; j_1 j_2 \dots j_p; *) = \prod_{k=2}^{p-1} R(i; j_1 j_k j_{k+1})$$



$$\rightarrow \quad \frac{Y_{ij}}{X_{ij}} \qquad \text{(superpropagator)}$$

- n, k correlator = sum over all graphs with n vertices, $n+k$ edges
- Bubbles vanish, need at least two edges ending on each vertex.
- Large N_c limit $\Rightarrow$ planar graphs

“NMHV” ($k = 1$) correlators all n



- Dotted edges denote an arbitrary number (or no) vertices
- sum over graphs of type C vanishes due to antisymmetry of the 3 vertex

Lightlike limit

In any (maximal or non-maximal) light like limit,

- only those graphs containing edges of the limit survive
- Surviving diagrams reduce to the Wilson loop diagrams
- Eg NMHV, maximal lightlike limit, only type A graphs with $\mu = 0$

Maximal Lightlike limit of NMHV correlator=NMHV amplitude:

$$\prod_{i=1}^{n} \frac{Y_i.Y_j}{X_i.X_j} \Big(\sum_{ij} R(i-1, i, j-1, j, *) \Big)$$

Next to maximal lightlike limit of NMHV correlator= 1 loop $n-1$ point MHV amplitude:

$$\prod_{i=1}^{n-1} \frac{Y_i.Y_j}{X_i.X_j} \Big(\sum_{ij} Kermit(i-1, i, j-1, j, *) \Big)$$

- More generally, correlator diagrams drawn on a sphere. Lightlike limit splits into two amplitude diagrams = Square of the WL

- Agrees with direct *x*-space component Feynman diagram computation. ✓

- Prove independence of spurious poles / $\mathcal{Z}_*$. (nearly ✓)

Conclusions and Future directions

- Integrand level we have four-point correlator up to 7 loops (using four-point amplitude)
- Conversely used correlator to obtain 5-point amplitude (up to 5 loops or 6 loops parity even)

$$\begin{array}{c}\text{Four-point amplitude}\\ \Downarrow \\ \text{Four-point correlator}\\ \Downarrow \\ \text{Five-point amplitude}\end{array}$$

- Higher-point MHV amplitude from four-point correlator (disentangle mixing from $(\text{NMHV})^2$??)
- We have found the integrals at three loops. Single valued multiple polylogs $\Rightarrow$ anomalous dimensions and three-point functions can be extracted. Integrability? [Drummond Duhr Eden Pennington Smirnov PH]

Conclusions and Future directions

- Derive twistor form from first principles.
  - Hidden symmetry type of approach succesful for 6point “NMHV” (= 5 point 1 loop) but not easily generalisable. Instead:
  - $R(i; j_1 j_2 j_3; *)$ as a new type of superconformal invariant depending on 4 analytic superspace points and one supertwistor: basis?
  - Unique *-independent combination?

- Link twistor approach and hidden symmetry aproach
  - First half, correlator constrains amplitude. Hidden symmetry.
  - Second half, lightlike limit automatic, no constraints. (Miracle of $*$ independence)
  - Clever choice of $\mathcal{Z}_*$ relates these two approaches?

- Amplituhedron, positivity etc.