Three-dimensional update

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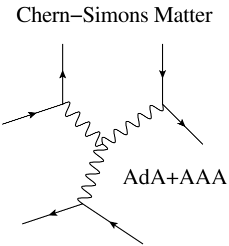
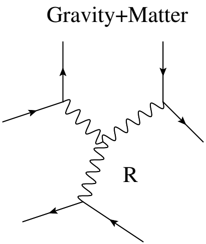
Prelude

Perturbation in a topological theory:





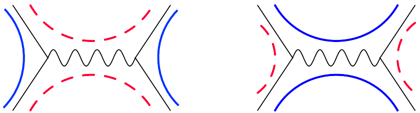
The suspect

Chenr-Simons Matter:

1. $\mathcal{N}=6$ (ABJM): $\mathrm{U(N)}_k \times \mathrm{U(N)}_{-k}$ gauge fields$(\mathrm{A}^\mu, \bar{\mathrm{A}}^\mu)$,

SU(4) bi-fundamental matter $(\phi^{\mathrm{I}}, \psi^{\mathrm{I}}, \bar{\phi}_{\mathrm{I}}, \bar{\psi}_{\mathrm{I}})$, $\mathrm{I}=1,2,3,4$

$$\mathcal{L} = \mathcal{L}_{C\,S} + \mathcal{L}_{\phi,Kin} + \mathcal{L}_{\psi,Kin} + \mathcal{L}_{4\phi^2\psi^2} + \mathcal{L}_{6\phi^6}$$



$$\begin{aligned}\Phi(\eta) &= \phi^4 + \eta^{\mathrm{I}}\psi_{\mathrm{I}} + \frac{1}{2}\epsilon_{\mathrm{IJK}}\eta^{\mathrm{I}}\eta^{\mathrm{J}}\phi^{\mathrm{K}} + \frac{1}{3!}\epsilon_{\mathrm{IJK}}\eta^{\mathrm{I}}\eta^{\mathrm{J}}\eta^{\mathrm{K}}\psi_4,\\ \bar{\Psi}(\eta) &= \bar{\psi}^4 + \eta^{\mathrm{I}}\bar{\phi}_{\mathrm{I}} + \frac{1}{2}\epsilon_{\mathrm{IJK}}\eta^{\mathrm{I}}\eta^{\mathrm{J}}\bar{\psi}^{\mathrm{K}} + \frac{1}{3!}\epsilon_{\mathrm{IJK}}\eta^{\mathrm{I}}\eta^{\mathrm{J}}\eta^{\mathrm{K}}\bar{\phi}_4,\end{aligned}$$

$$\mathcal{A}_n(\bar{1}2\bar{3}\cdots n)(\lambda,\eta)$$
$$\mathcal{A}_n(1\bar{2}3\cdots \bar{n})(\lambda,\eta)$$

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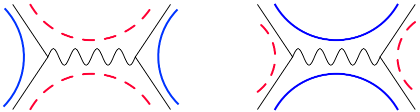
$$\mathcal{L} = \mathcal{L}_{CS} + \mathcal{L}_{\phi,Kin} + \mathcal{L}_{\psi,Kin} + \mathcal{L}_{4\phi^2\psi^2} + \mathcal{L}_{6\phi^6}$$



2. $\mathcal{N}=8$ (BLG): $\mathrm{SU(2)}_k \times \mathrm{SU(2)}_{-k}$ gauge fields$(\mathrm{A}^\mu, \bar{\mathrm{A}}^\mu)$,
SO(8) adjoint matter $(\phi^{\mathrm{I_v}}, \psi^{\mathrm{I_c}})$

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d$$

The suspect

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$$[T^a, T^b, T^c] = f^{abc}{}_d T^d$$

Super Gravity:
1. $\mathcal{N} = 16$ supergravity: Marcus, Schwarz
128 scalars are in the spinor representation of SO(16) $\in E_{8,8}/\mathrm{SO}(16)$
$\rightarrow \mathcal{M}_n = 0$ for odd n

The gauge theory: ABJM

Prelude

The known for $\mathcal{N} = 4$ SYM:

- The planar theory enjoys SU(2,2|4) DSCI
- The string sigma model enjoys fermionic self T-duality
- The (super)amplitude is dual to a (super)Wilson-loop
- The IR-divergence structure captured by BDS
- The leading singularities is given by residues of $Gr(k, n)$
- The amplitude has uniform trancsendentality
- The amplitudehedron

Prelude

As comparison to ABJM:

- The planar theory enjoys SU(2,2|4) DSCI $\rightarrow$ OSp(6|4)
- The IR-divergence structure captured by BDS $\rightarrow$ Remarkably yes
- The leading singularities is given by residues of $Gr(k, n)$ $\rightarrow$ OG(k,2k)
- The amplitude has uniform trancsendentality (*) $\rightarrow$ True so far

Known unknowns:

1. Why is the IR-divergence (Dual conformal anomaly equation) the same? Y-t, W. Chen, S. Caron-Huot

$$\mathcal{A}_4^{2-\text{loop}} = \left(\frac{N}{k}\right)^2 \frac{\mathcal{A}_4^{\text{tree}}}{2} \text{BDS}_4$$

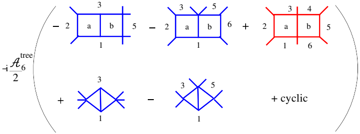
$$\mathcal{A}_6^{2-\text{loop}} = \left(\frac{N}{k}\right)^2 \left\{ \frac{\mathcal{A}_6^{\text{tree}}}{2}\left[\text{BDS}_6 + R_6\right] + \frac{\mathcal{A}_{6,\text{shifted}}^{\text{tree}}}{4i}\left[\ln\frac{u_2}{u_3}\ln\chi_1 + \text{cyclic}\times 2\right]\right\}$$

At four-point to all orders in ϵ M. Bianchi, M. Leoni, S Penati, exponentiation verified at three-loops M. Bianchi, M. Leoni

Known unknowns: 2. Why is the amplitude non-analytic?

$$\mathcal{A}_6^{\text{1-loop}} = \frac{\mathcal{A}_6^{\text{tree}}}{\sqrt{2}}\left[I_{box}(3,4,5,1) + I_{box}(1,2,3,4) - I_{box}(4,5,6,1) - I_{box}(6,1,2,4)\right] + \frac{\mathcal{C}_1+\mathcal{C}_1^*}{2} I_{tri}(1,3,5) + \frac{\mathcal{C}_2+\mathcal{C}_2^*}{2} I_{tri}(2,4,6)\,.$$

$$\rightarrow \quad \mathcal{A}_6^{\text{1-loop}} = \binom{N}{k}\frac{-\pi}{2}\mathcal{A}^{\text{tree}}_{6,\text{shifted}}\left(\text{sgn}_c\langle 12\rangle\text{sgn}_c\langle 34\rangle\text{sgn}_c\langle 56\rangle + \text{sgn}_c\langle 23\rangle\text{sgn}_c\langle 45\rangle\text{sgn}_c\langle 61\rangle\right).$$



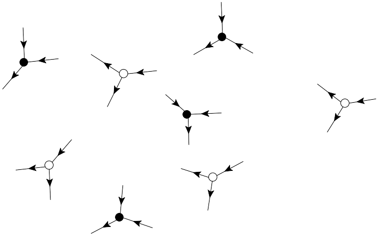
$$\rightarrow \quad \mathcal{A}_6^{\text{2-loop}} = \binom{N}{k}^2\left\{\frac{\mathcal{A}_6^{\text{tree}}}{2}\left[BDS_6 + R_6\right] + \frac{\mathcal{A}^{\text{tree}}_{6,\text{shifted}}}{2}\times \left[\text{sgn}_c(\langle 12\rangle)\text{sgn}_c(\langle 45\rangle)\frac{(\langle 34\rangle\langle 46\rangle + \langle 35\rangle\langle 56\rangle)}{\sqrt{(\langle 34\rangle\langle 46\rangle + \langle 35\rangle\langle 56\rangle)^2}}\log\frac{u_2}{u_3}\arccos(\sqrt{u_1}) + \text{cyclic}\times 2\right]\right\}$$

Unknown knowns

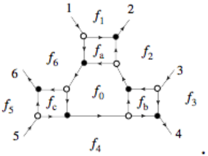
- The string sigma model enjoys fermionic self T-duality $\rightarrow$ Unsucessful
- The (super)amplitude is dual to a (super)Wilson-loop $\rightarrow$ Unsucessful

Prelude

Planar $\mathcal{N} = 4$ SYM $\in \mathrm{Gr}(\mathrm{k},\mathrm{n})_+$ Arkani-Hamed, J. Bourjaily, F. Cachazo, A. Goncharov, A. Postnikov, J. Trnka



$\rightarrow$



$\rightarrow \quad C_{\alpha i}(f)$

$$C_{\alpha i}(f) = \begin{pmatrix} 1 & \frac{1}{f_1} + \frac{1}{f_1 f_a (1+f_0)} & 0 & \frac{f_4 f_5 f_6 f_c}{1+1/f_0} & 0 & \frac{f_6}{1+1/f_0} \\ 0 & \frac{f_2}{1+1/f_0} & 1 & \frac{1}{f_3} + \frac{1}{f_3 f_b (1+f_0)}, & 0 & \frac{f_1 f_2 f_6 f_a}{1+1/f_0} \\ 0 & \frac{f_3 f_4 f_2 f_b}{1+1/f_0} & 0 & \frac{f_4}{1+1/f_0} & 1 & \frac{1}{f_5} + \frac{1}{f_5 f_c (1+f_0)} \end{pmatrix}$$

$$\mathcal{A}_n = \sum_{\mathrm{dia}} \int \prod_i \frac{\mathrm{df_i}}{\mathrm{f_i}} \delta^{4k|4k} (\mathrm{C} \cdot \mathcal{W})$$

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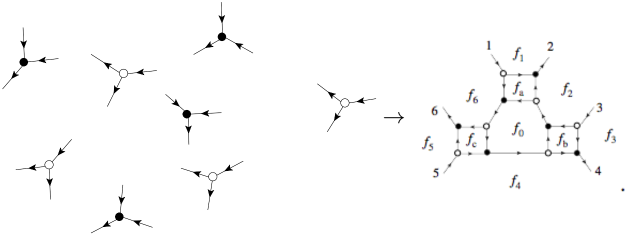

$\rightarrow \mathrm{C}_{\alpha\mathrm{i}}(\mathrm{f})$

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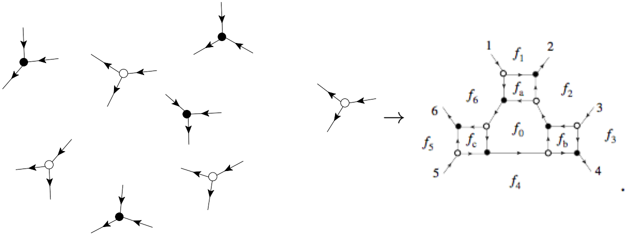

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Conclusion

- The scattering amplitude of ABJM is given by integrals over cells in the positive orthogonal grassmannian $\mathrm{OG}_{\mathrm{k}+}$
- Each cell in the positive orthogonal grassmannian $\mathrm{OG}_{\mathrm{k}+} \rightarrow$ cell $\mathrm{Gr}(\mathrm{k}, 2\mathrm{k})_+$.
- The canonical form has logarithmic singularity at $\partial\mathrm{OG}_{\mathrm{k}+}$

Orthogonal Grasmmannian

Consider k-planes in n-dimensional space equipped with a symmetric bi-linear Q^{ij}

The orthogonal grassmannian $\equiv \mathrm{Q^{ij}C_{\alpha i}C_{\beta j} = 0}$

Consider $\mathrm{n = 2k}$ and $\mathrm{Q^{ij}} = \eta^{\mathrm{ij}}$ signature $(+,+,+,\cdots,+)$

$$
\begin{aligned}
k &= 1, & \mathrm{C_{\alpha i}} &= (1, \pm \mathrm{i}) \\
k &= 2, & \mathrm{C_{\alpha i}} &= \begin{pmatrix} 1 & \pm i\cos z & 0 & -i\sin z \\ 0 & \pm i \sin z & 1 & i\cos z \end{pmatrix}
\end{aligned}
$$

$$
\mathrm{A_n^{tree}} = \sum_{\mathrm{res}} \int \frac{\mathrm{dC}}{(1\cdots \mathrm{k})\cdots(\mathrm{k}\cdots \mathrm{n}-1)} \delta(\mathrm{Q^{ij}C_{\alpha i}C_{\beta j}})\delta^{2\mathrm{k}}(\mathrm{C}\cdot\lambda)\delta^{3\mathrm{k}}(\mathrm{C}\cdot\eta)
$$

S. Lee, D. Gang, E. Koh, E. Koh, A. Lipstein, Y-t

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$$k = 2, \qquad C_{\alpha i} = \begin{pmatrix} 1 & \pm i\cos z & 0 & -i\sin z \\ 0 & \pm i\sin z & 1 & i\cos z \end{pmatrix}$$

$$A_n^{tree} = \sum_{res} \int \frac{dC}{(1\cdots k)\cdots(k\cdots n-1)} \delta(Q^{ij}C_{\alpha i}C_{\beta j})\delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$

S. Lee, D. Gang, E. Koh, E. Koh, A. Lipstein, Y-t

Positive Orthogonal Grasmmannian

Positivity: $(\mathrm{i}, \mathrm{i}+1, \cdots, \mathrm{i}+\mathrm{k}) > 0$

$\mathrm{Q}^{\mathrm{ij}} = \eta^{\mathrm{ij}}$ signature $(+, +, +, \cdots, +)$

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Positive Orthogonal Grasmmannian

Positivity: ordered $(\mathrm{i}, \cdots, \mathrm{j}) > 0$

$\mathrm{Q}^{\mathrm{ij}} = \eta^{\mathrm{ij}}$ signature $(+, -, +, \cdots, -)$

$$k = 1, \qquad \mathrm{C}_{\alpha\mathrm{i}} = (1, 1)$$

$$k = 2, \qquad \mathrm{C}_{\alpha\mathrm{i}} = \begin{pmatrix} 1 & \cos \mathrm{z} & 0 & -\sin \mathrm{z} \\ 0 & \sin \mathrm{z} & 1 & \cos \mathrm{z} \end{pmatrix}$$

Positive for $0 \leq \mathrm{z} \leq \pi/2$

Volume form w. logarithmic singularity at the boundary: $z = \pi/2$, $z = 0$

$$\frac{\mathrm{dz}}{\cos \mathrm{z} \sin \mathrm{z}} = \mathrm{d} \log \tan \mathrm{z}$$

$$\int \mathrm{d} \log \tan \quad \delta^4(\mathrm{C} \cdot \lambda) \delta^6(\mathrm{C} \cdot \eta)$$

This is not the amplitude $\mathcal{A}_4$!

Positive Orthogonal Grasmmannian

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Branches of Positive Orthogonal Grasmmannian

$$k = 2, \qquad C_{\alpha i} = \begin{pmatrix} 1 & \cos z & 0 & -\sin z \\ 0 & \sin z & 1 & \cos z \end{pmatrix}$$

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For $0 \leq z \leq \pi/2$ Positivity: $(i, \cdots, j) > 0$ and $\pm(i, \cdots, 2k) > 0$

$$\mathcal{A}_4 = \int d \log \tan \quad \delta^4(C \cdot \lambda)\delta^6(C \cdot \eta) + (\overline{OG}_{2+})$$

The four-point amplitude is given by the sum of two branches in OG_{2+}

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Why Two Branches of Positive Orthogonal Grasmmannian

$$k=2, C_{\alpha i}=\begin{pmatrix} 1 & \cos z & 0 & -\sin z \\ 0 & \sin z & 1 & \cos z \end{pmatrix}$$

$$\delta^4(C\cdot\lambda)\rightarrow \begin{matrix} \lambda_1+\cos z\lambda_2-\sin z\lambda_4=0 \\ \lambda_3+\sin z\lambda_2+\cos z\lambda_4=0 \end{matrix} \rightarrow \langle 34\rangle=\langle 12\rangle$$

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$$\delta^4(C\cdot\lambda)\rightarrow \begin{matrix} \lambda_1+\cos z\lambda_2+\sin z\lambda_4=0 \\ \lambda_3+\sin z\lambda_2-\cos z\lambda_4=0 \end{matrix} \rightarrow \langle 34\rangle=-\langle 12\rangle$$

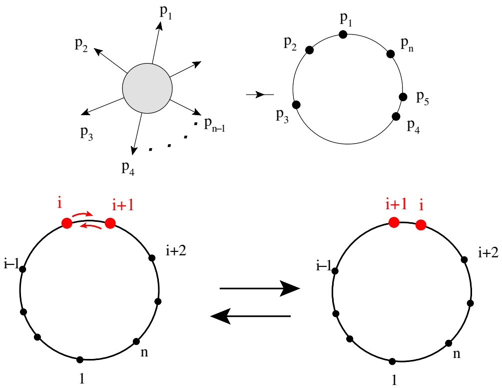
There are two branches in the kinematics as well:

$$\langle 34\rangle^2=s_{34}=s_{12}=\langle 12\rangle^2$$

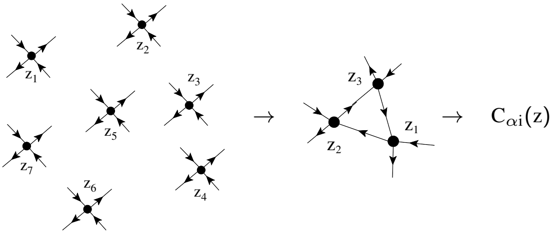
Why Two Branches of Positive Orthogonal Grasmmannian

3D- kinematics is topologically a circle

$p_i = (1, \cos\theta_i, \sin\theta_i)$



On-shell diagrams in Orthogonal Grasmmannian



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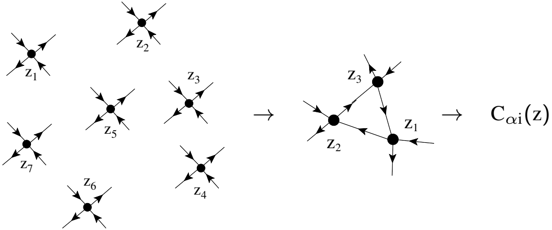
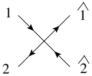
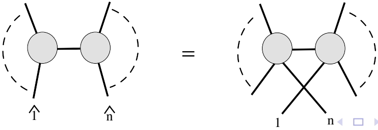


1. Are these diagrams related to $\mathcal{A}_n$? Arkani-Hamed, J. Bourjaily, F. Cachazo, A. Goncharov, A. Postnikov, J. Trnka



$$\delta^4(C\cdot\lambda) \rightarrow \begin{array}{l} \lambda_{\hat{1}} + \sec z\lambda_1 + \tan z\lambda_2 \\ \lambda_{\hat{2}} - \tan z\lambda_1 - \sec z\lambda_2 \end{array}$$



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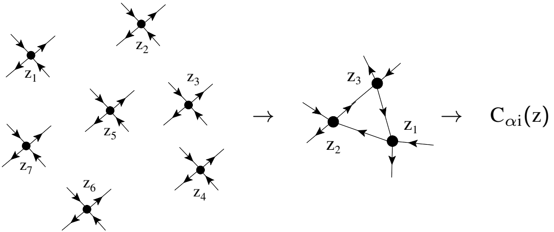
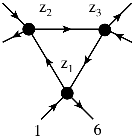


1. Are these diagrams related to $\mathcal{A}_n$?

$$\mathcal{A}_6 = \sum_{\text{branch}} \int d\log\tan_1\, d\log\tan_2\, d\log\tan_3\, \delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$



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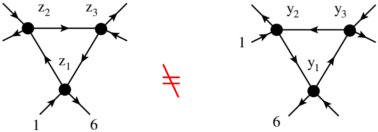
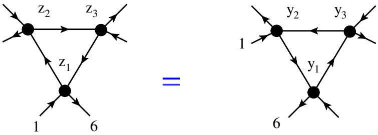
No



1. Are these diagrams related to $\mathcal{A}_n$?

$$\mathcal{A}_6 = \sum_{\text{branch}} \int \mathrm{d}\log\tan_1 \mathrm{d}\log\tan_2 \mathrm{d}\log\tan_3 (1 + \sin_1 \sin_2 \sin_3)\delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$

Yes



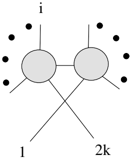
$$\mathcal{A}_6 = \sum_{\text{branch}} \int \mathrm{d}\log\tan_1 \mathrm{d}\log\tan_2 \mathrm{d}\log\tan_3 (1 + \cos_1 \cos_2 \cos_3)\delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$

No new singularities $0 \leq z \leq \pi/2$.

On-shell diagrams in Orthogonal Grasmmannian

In general

$$\sum_{i}$$

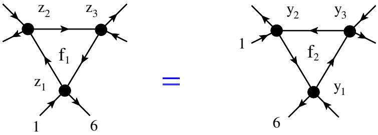


$$\mathcal{A}_n = \sum_{\text{branch}} \sum_{\text{dia}} \int \prod_{i=1}^{k} \mathrm{d}\log\tan_i \, \mathcal{J} \delta^{2k}(C \cdot \lambda) \delta^{3k}(C \cdot \eta)$$

How to get $\mathcal{J}$?

On-shell diagrams in Orthogonal Grasmmannian

$$\mathcal{A}_6 = \sum_{\text{branch}} \int \mathrm{d}\log \tan_1 \mathrm{d}\log \tan_2 \mathrm{d}\log \tan_3 (1 + \sin_1 \sin_2 \sin_3)\delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$



$$\mathcal{A}_6 = \sum_{\text{branch}} \int \mathrm{d}\log \tan_1 \mathrm{d}\log \tan_2 \mathrm{d}\log \tan_3 (1 + \cos_1 \cos_2 \cos_3)\delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$

$\mathcal{J}$ is naturally associated with faces!

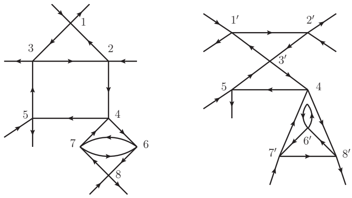
On-shell diagrams in Orthogonal Grasmmannian

$$\mathcal{J} = 1 + \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 + \mathcal{J}_{13} + \mathcal{J}_{23}$$

- $\mathcal{J}_1$:

$$\mathcal{J}_1 = \sum_{\text{single}} J_i + \sum_{\text{disjoint pairs}} J_i J_j + \sum_{\text{disjoint triples}} J_i J_j J_k + \dots$$

- $\mathcal{J}_2$: Two closed loops sharing a single vertex
- $\mathcal{J}_3$: Two closed loops sharing two vertices without sharing an edge.
- $\mathcal{J}_{13}$ and $\mathcal{J}_{23}$: The effect of the bigger loop from $\mathcal{J}_3$.

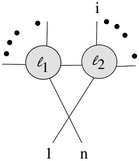


(a) (b)

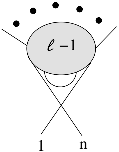
Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

The loop-level recurssion Arkani-Hamed, J. Bourjaily, F. Cachazo, A. Goncharov, A. Postnikov, J. Trnka

$$\mathcal{A}_n^l = \sum_{l_1+l_2=l} \sum_{i=4}^{n-2}$$

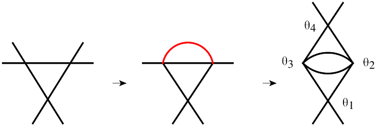


$+$



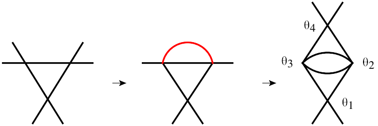
Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

The loop-level recurssion Arkani-Hamed, J. Bourjaily, F. Cachazo, A. Goncharov, A. Postnikov, J. Trnka

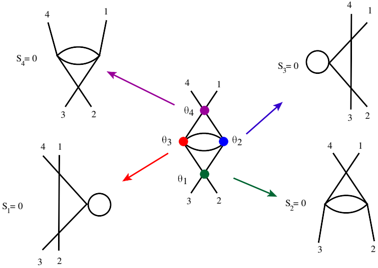


$= \quad \mathcal{A}_4^{\ell=1}$

The loop-level recurssion Arkani-Hamed, J. Bourjaily, F. Cachazo, A. Goncharov, A. Postnikov, J. Trnka

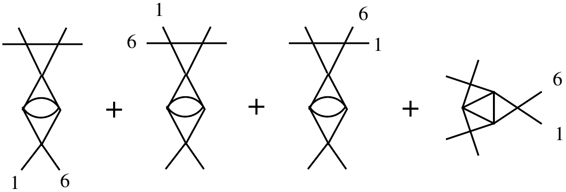


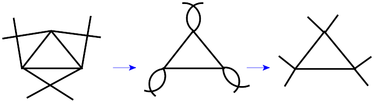
$$= \mathcal{A}_4^{\ell=1}$$



Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

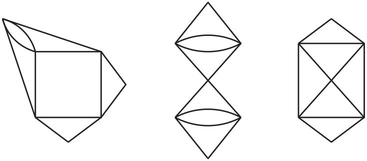
$$A_6^{1-\text{loop}} =$$





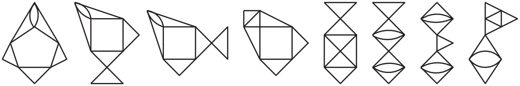
Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

$A_4^{2-\text{loop}} =$



$+ \ (i \to i{+}2)$

$A_6^{2-\text{loop}} =$

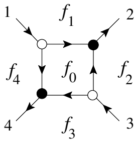


$+ \ (i \to i{+}2)$

Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

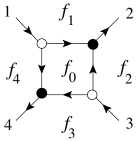
- The solution to BCFW is manifestly cylic $i \to i+2$
- For each cell, a single chart covers all singularities
- All loop: 4 and 6-point amplitudes is a product of independent $d\log$
- Proved all physical sing present, spurious cancels

Embedding $\mathrm{OG}(k, 2k)$ into $\mathrm{G}(k, 2k)$



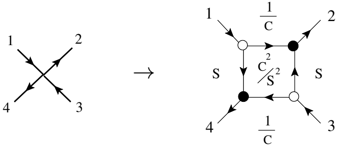
$$C = \begin{pmatrix} 1 & 1/f_1 & 0 & -f_4 \\ 0 & f_2 & 1 & 1/f_3 \end{pmatrix} .$$

Embedding $\mathrm{OG}(k,2k)$ into $\mathrm{G}(k,2k)$



$$C = \begin{pmatrix} 1 & 1/f_1 & 0 & -f_4 \\ 0 & f_2 & 1 & 1/f_3 \end{pmatrix}.$$

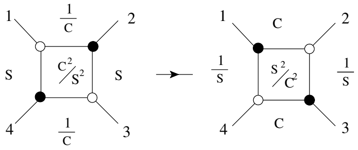
$$f_1 = \frac{1}{c},\ f_4 = s,\ f_2 = s,\ f_3 = \frac{1}{c}$$



OG_{2+} has an image in $Gr(2,4)_+$

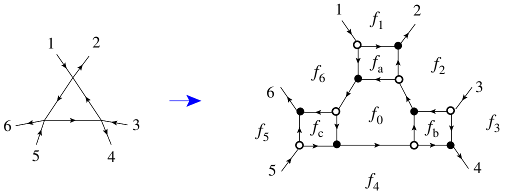
$$C = \begin{pmatrix} 1 & 1/f_1 & 0 & -f_4 \\ 0 & f_2 & 1 & 1/f_3 \end{pmatrix} .$$

Cluster transformation:



$c, s \rightarrow \frac{1}{c}, \frac{1}{s}$

Embedding $\mathrm{OG}(k, 2k)$ into $\mathrm{G}(k, 2k)$



$$(f_a, f_b, f_c) = (c_1^2/s_1^2, c_2^2/s_2^2, c_3^2/s_3^2),\ f_0 = \frac{1}{c_1 c_2 c_3}$$

$$f_1 = \frac{1}{c_1},\ f_2 = s_1 s_2,\ f_3 = \frac{1}{c_3},\ f_4 = s_2 s_3,\ f_5 = \frac{1}{c_3},\ f_6 = s_1 s_3$$

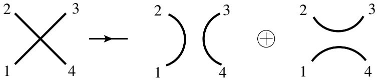
- The variable for the k new faces is simply $f = c^2/s^2$.
- Take a clockwise orientation on each face. The contribution from each vertex is $1/c$ if one first encounters the black vertex, otherwise the contribution is s.

The combinatorics of the cells in Orthogonal Grasmmannian

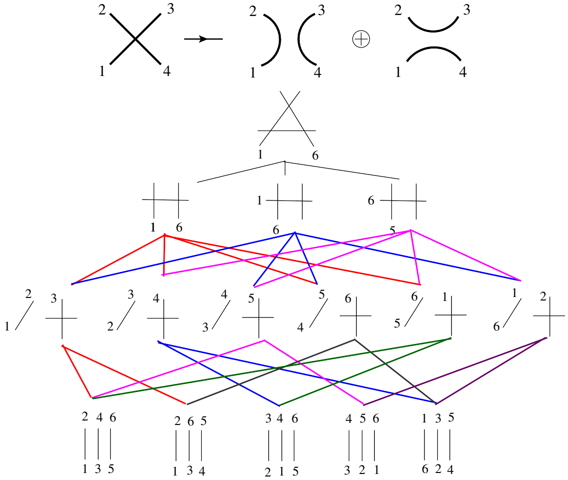
$$k = 2, \qquad C_{\alpha i} = \begin{pmatrix} 1 & \cos z & 0 & -\sin z \\ 0 & \sin z & 1 & \cos z \end{pmatrix}$$

Volume form w. logarithmic singularity at the boundary: $z = \pi/2$, $z = 0$

$$\frac{dz}{\cos z \sin z} = d \log \tan z$$

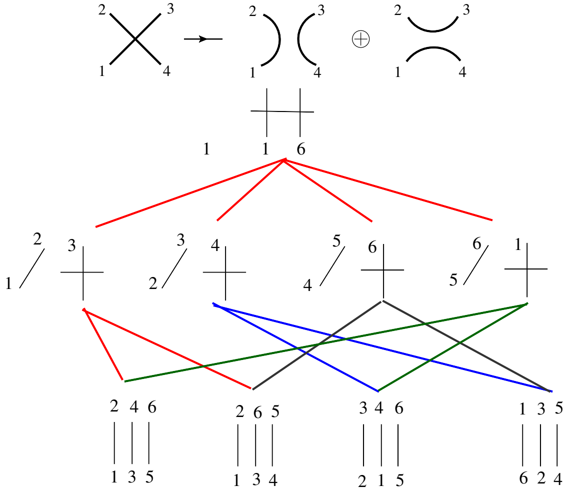


The combinatorics of the cells in Orthogonal Grasmmannian



$$-1+3-6+5=1$$

The combinatorics of the cells in Orthogonal Grasmmannian



$$1 - 4 + 4 = 1$$

The combinatorics of the cells in Orthogonal Grasmmannian

A generating function for the number of cells J. Kim, S. Lee

$$T_k(q) = \sum_{l=0}^{k(k-1)/2} T_{k,l} q^l = \frac{1}{(1-q)^k} \sum_{j=-k}^{k} (-1)^j \left(\begin{array}{c} 2k \\ k+j \end{array} \right) q^{j(j-1)/2} \quad (1)$$

$l =$ number of vertices. For top-cells the Euler number is always 1

$$T_k(-1) = \sum_{l=0}^{k(k-1)/2} T_{k,l}(-1)^l = 1$$

Poset is Eulerian Thomas Lam

Momentum Twistors

In four-dimensions:

$$Y_i^{AB} = Z_i^{[A} Z_{i-1}^{B]}$$

$$Y^{AB}\Omega_{AB} = 0\,,\Omega_{AB} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}.$$

This also implies that the bi-twistors Z_i must satisfy:

$$Z_i^A Z_{i+1}^B \Omega_{AB} = 0\,.4$$

With $p_i = E_i(1, \sin\theta_i, \cos\theta_i)$, where E_i is the energy

$$\lambda_{ia} = \begin{pmatrix} -\sin\frac{\theta_i}{2} \\ \cos\frac{\theta_i}{2} \end{pmatrix}, \qquad \tilde{\lambda}_{ia} = -2E_i\lambda_{ia}$$

Such that we still have $p_i = \lambda_i\tilde{\lambda}_i$

$$\mu_i^a = y_i^{ab}\lambda_{ib} = y_{i+1}^{ab}\lambda_{ib}$$

Momentum Twistors

The momentum-twistor space grassmannian

$$\mathcal{L}_{n;k} = J \times \delta^3(P)\,\delta^6(\mathcal{Q}) \times \int \frac{d^{kn}C}{GL(k)} \frac{\delta^{\frac{k(k+1)}{2}}\left(C_{\hat{\alpha}i}G^{ij}C_{\hat{\beta}j}\right)\delta^{4k|3k}(C\cdot\mathcal{Z})}{M_2 M_3 \cdots M_{k+3}},$$

Where the metric is given as:

$$\begin{aligned} G^{i,i+2} &= \tfrac{k}{n}[i,i{+}2] \\ G^{i,i+3} &= \tfrac{k-1}{n}[i,i{+}3] \\ &\vdots \\ G^{i,i+k+1} &= \tfrac{1}{n}[i,i{+}k{+}1] \end{aligned}$$

where $[i,j] \equiv Z_i^A Z_j^B \Omega_{AB}$.

In momentum twistor space: an orthogonal grassmannian with kinematic dependent metric.

Locality

The NMHV residues in $\mathcal{N}=4$ SYM

$$\int \prod_i \frac{dc_i}{c_i} \delta^{4|4}(Z_1 + c_i \mathcal{Z}_i)\,.$$

The singularity $c_i = 0$ correspond to $\langle 1, j, k, l \rangle = 0$. The integral we wish to study is now

$$\int \prod_{i=2}^{6} \frac{dc_i}{c_i} \delta(c_i G^{ij} c_j) \delta^{4|4}(Z_1 + \sum_{i=2}^{6} c_i \mathcal{Z}_i)\,. \qquad (2)$$

To analyze the singularity of this integral, set $c_2 = 0$. Note that since all of the c_is are already fixed by the bosonic delta functions, setting $c_2 = 0$ can only be allowed if there is extra constraint on the external data. With $c_2 = 0$, the orthogonal constraint now requires

$$[1,3]c_3 + c_3[3,5]c_5 + c_4[4,6]c_6 + c_5[5,1] = 3c_3[3,5]c_5$$

For $c_5 = 0$, $C \cdot Z = 0$ simply fixes the remaining 3 c_is, and the condition $\langle 3461 \rangle = 0$.

The orthogonal constraint is such that only local poles are allowed!

Locality

The NMHV residues in $\mathcal{N}=4$ SYM

$$\int \prod_i \frac{dc_i}{c_i}\delta^{4|4}(Z_1+c_i\mathcal{Z}_i)\,.$$

The singularity $c_i=0$ correspond to $\langle 1,j,k,l\rangle=0$.

$$\int \prod_{i=2}^{6} \frac{dc_i}{c_i}\delta(c_iG^{ij}c_j)\delta^{4|4}(Z_1+\sum_{i=2}^{6}c_i\mathcal{Z}_i)\,. \tag{3}$$

The orthogonal constraint is such that only local poles are allowed!

$$\sum_{s=\pm}\frac{[24]^2\delta^{(3)}(c^{(s)}\cdot\eta)\left[[35](\langle 1236\rangle\langle 2456\rangle-\langle 5612\rangle\langle 2346\rangle)+s\sqrt{D}([26][35]+[25][36])\right]}{\langle 1234\rangle\langle 5612\rangle\langle 1245\rangle\langle 3461\rangle\langle 2356\rangle^3}\,. \tag{4}$$

with $D=\prod_{i=1}^{6}[i\,i+2]$

The gravity theory: $\mathcal{N} = 16$ SUGRA

The fact that $\mathcal{M}_n = 0$ for odd n:

- Pure 3D SUGRA amplitude is very different than closed string amplitudes.
- Unlike 4D, the duality group acts on ALL states in the theory$\rightarrow$ double soft limits
  W, Chen, C. Wen, Y-t

$$M_{n+2}|_{\eta^{10}} \sim \frac{(p_1 - p_2)}{p_i \cdot (p_1 + p_2)} \eta_i \eta_i M_n$$

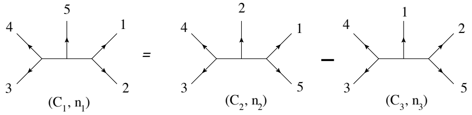
$$M_{n+2}|_{\eta^{8}} \sim \frac{(p_1 - p_2)}{p_i \cdot (p_1 + p_2)} \eta_i \frac{\partial}{\partial \eta_i} M_n$$

$$M_{n+2}|_{\eta^{6}} \sim \frac{(p_1 - p_2)}{p_i \cdot (p_1 + p_2)} \frac{\partial}{\partial \eta_i} \frac{\partial}{\partial \eta_i} M_n$$

- Gravity=(YM)^2 is a redundant description.
- How can Gravity=(CSm)^2 work (No string theory)

BCJ to the rescue: Bern-Carrasco-Johansson(BCJ): Duality between color and kinematics for (super)Yang-Mills:

$$A_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i n_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$



$$n_i = (k_4 \cdot k_5)(k_3 \cdot \epsilon_1)(\epsilon_2 \cdot \epsilon_3)(\epsilon_4 \cdot \epsilon_5) + \cdots$$

$$c_1 = f^{\,34a} f^{\,a5b} f^{\,b12}, \quad c_2 = f^{\,34a} f^{\,a2b} f^{\,b15}, \quad c_3 = f^{\,34a} f^{\,a1b} f^{\,b25}$$

$$\boxed{c_1 = c_2 - c_3 \leftrightarrow n_1 = n_2 - n_3}$$

$$[T_a, [T_b, T_c]] + [T_b, [T_c, T_a]] + [T_c, [T_a, T_b]] = 0$$

Consequences

$$c_1 = c_2 - c_3 \leftrightarrow n_1 = n_2 - n_3$$

$$s_{24}\, A(1,2,4,3,5) = (s_{14} + s_{45})\, A(1,2,3,4,5) + s_{14}\, A(1,2,3,5,4)$$

Once a BCJ n_i is found, one obtains gravity:

$$\text{YM}: \quad A_n^{\text{tree}} = g^{n-2} \sum_i \frac{n_i c_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

$$\text{Gravity}: \quad M_n^{\text{tree}} = i \left(\frac{\kappa}{2}\right)^{n-2} \sum_i \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

Most importantly, the same for loops:

$$\frac{(-i)^L}{g^{n-2+2L}} A_n^L = \sum_j \int \prod_{\ell=1}^{L} \frac{d^D p_\ell}{(2\pi)^D} \frac{c_j n_j}{S_j \prod_{\alpha_j} p_{\alpha_j}^2}$$

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}} M_n^L = \sum_j \int \prod_{\ell=1}^{L} \frac{d^D p_\ell}{(2\pi)^D} \frac{\tilde{n}_j n_j}{S_j \prod_{\alpha_j} p_{\alpha_j}^2}$$

Proven to all order in perturbation theory: Bern, Dennen, Kiermiaer, Y-T

Double double copy in 3D

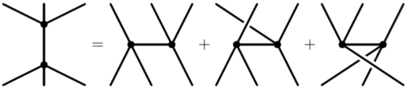
New color-kinematic dualities:

N=8 Chern-Simons-Matter theory

3-algebra gauge group $[T^a, T^b, T^c] = f^{abc}{}_d T^d$ Bagger, Lambert, Gustavsson

Obeys color-kinematics duality. Bargheer, He and McLoughlin

Fundamental identity (Jacobi identity):



$$c_s = c_t + c_u + c_v \Leftrightarrow n_s = n_t + n_u + n_v$$

4 and 6 point checks shows that the double copy of BLG Is N = 16 $E_{8(8)}$ SG of Marcus and Schwarz

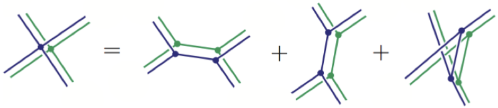
Bargheer, He and McLoughlin

BLG ='square root' of *N*=16 SG $A_4^{\text{BLG}} = \sqrt{M_4^{\mathcal{N}=16}} = \sqrt{\frac{\delta^{16}(Q)}{stu}}$

Double double copy in 3D

Duality between Color-Kinematic Dualities: $(\text{Lie 2 Algebra})^2 = (\text{Lie 3 Algebra})^2$

In *D*=3, supergravity obtained from in two different double copies:



CSM ⊗ CSM = SYM ⊗ SYM (kinematic parts) **Huang, H.J.**

- **The extra propagators in SYM ⊗ SYM compensates for dimension mismatch**

Double double copy in 3D

Duality between Color-Kinematic Dualities: $(\text{Lie 2 Algebra})^2 = (\text{Lie 3 Algebra})^2$

Summary of verified double copy constructions: Huang, H.J.

SG theory	$\text{CSm}_L \times \text{CSm}_R$ = supergravity	$\text{sYm}_L \times \text{sYM}_R$ = supergravity	coset
$\mathcal{N}=16$	$16^2 = 256$	$16^2 = 256$	$E_{8(8)}/SO(16)$
$\mathcal{N}=12$	$8^2+\bar{8}^2 = 16\times(4+\bar{4}) = 128$	$16\times 8 = 128$	$E_{7(-5)}/SO(12)\otimes SO(3)$
$\mathcal{N}=10$	$8\times 4+\bar{8}\times\bar{4} = 16\times(2+\bar{2}) = 64$	$16\times 4 = 64$	$E_{6(-14)}/SO(10)\otimes SO(2)$
$\mathcal{N}=8,\ n=2$	$4^2+\bar{4}^2 = 8\times 2+\bar{8}\times\bar{2} = 32$	$16\times 2 = 32$	$SO(8,2)/SO(8)\otimes SO(2)$
$\mathcal{N}=8,\ n=1$	$16\times 1 = 16$	$16\times 1 = 16$	$SO(8,1)/SO(8)$

Examples 4pts:

$$\mathcal{M}_4^{\mathcal{N}=12}(\bar{1},2,\bar{3},4) = (A_4^{\mathcal{N}=6})^2 = \left(\frac{\delta^{(6)}(\sum_i \lambda^\alpha \eta_i^I)}{\langle 1\,2\rangle\langle 2\,3\rangle}\right)^2$$

$$\mathcal{M}_{4,n=2}^{\mathcal{N}=8}(\bar{1},2,\bar{3},4) = (A_4^{\mathcal{N}=4})^2 = \left(\frac{\delta^{(4)}(\sum_i \lambda^\alpha \eta_i^I)\langle 1\,3\rangle}{\langle 1\,2\rangle\langle 2\,3\rangle}\right)^2$$

$$\mathcal{M}_{4,n=1}^{\mathcal{N}=8} = \frac{1}{2}\frac{\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^I)(s^2+t^2+u^2)}{\langle 1\,2\rangle^2\langle 2\,3\rangle^2\langle 1\,3\rangle^2}$$

checked double copy up to 6pts!

New amplitude relations

In principle, any partial amplitude is proportional to any other, but...
amplitude relations important to study -> gives clues to the C-K structure

$$\mathcal{A}_m = \sum_{i \in \text{quartic}} \frac{n_i c_i}{\prod_{\alpha_i} s_{\alpha_i}} \quad \xrightarrow{\text{Solve Jacobi}} \quad A_{(i)} = \sum_{j=1}^{p} \Theta_{ij} n_j$$

At 6pts (ABJM):

$$\Theta_{ij} = \begin{pmatrix} \frac{1}{s_1} & \frac{1}{s_2}+\frac{1}{s_9} & \frac{1}{s_9} & -\frac{1}{s_9} & 0 \\ \frac{1}{s_8} & -\frac{1}{s_8} & \frac{1}{s_3} & \frac{1}{s_4}+\frac{1}{s_8} & 0 \\ \frac{1}{s_7} & -\frac{1}{s_7} & -\frac{1}{s_6}-\frac{1}{s_7} & \frac{1}{s_6}+\frac{1}{s_7} & \frac{1}{s_5}+\frac{1}{s_6}+\frac{1}{s_7} \\ 0 & -\frac{1}{s_9} & -\frac{1}{s_3}-\frac{1}{s_9} & \frac{1}{s_9} & -\frac{1}{s_5} \\ 0 & -\frac{1}{s_2} & \frac{1}{s_6} & -\frac{1}{s_4}-\frac{1}{s_6} & -\frac{1}{s_6} \end{pmatrix}$$

5×5 matrix has rank 4, but only in *D*=3 and on-shell !

5-term amplitude relation:

$$\text{Ker}(\Theta^T) \cdot A = \sum_{i=1}^{5} C_{ik} A_{(i)} = 0$$

$$\text{Det}(\Theta_{i1}, \Theta_{i2}, \ldots, A_{(i)}, \ldots, \Theta_{ip}) = 0$$

Trouble beyond six-point

Beyond six-points: No new amplitude relations for $\mathcal{N} < 8$!

Amplitude relations for BLG up to 12 points, and squares to $\mathcal{N} = 16$ SUGRA.

Why not ABJM? General gauge invariance

$$\Delta_i = \Delta p^2_{\alpha_i}, \quad \sum_i \frac{\Delta_i c_i}{\prod_{\alpha_i} p^2_{\alpha_i}} = 0$$

ABJM partial amplitudes are not invariant under the BLG general gauge invariance

$$\begin{aligned}
A^{\mathrm{BLG}}(\bar{1}\bar{3}\bar{5}\bar{7}2;468) &= A^{\mathrm{ABJM}}(\bar{1}4\bar{3}6\bar{5}8\bar{7}2) + A^{\mathrm{ABJM}}(\bar{1}4\bar{3}8\bar{5}6\bar{7}2) + A^{\mathrm{ABJM}}(\bar{1}6\bar{3}4\bar{5}8\bar{7}2) \\
&\quad + A^{\mathrm{ABJM}}(\bar{1}6\bar{3}8\bar{5}4\bar{7}2) + A^{\mathrm{ABJM}}(\bar{1}8\bar{3}4\bar{5}6\bar{7}2) + A^{\mathrm{ABJM}}(\bar{1}8\bar{3}6\bar{5}4\bar{7}2) \\
A^{\mathrm{BLG}}(\bar{3}2\bar{5}\bar{7}8;\bar{1}46) &= A^{\mathrm{ABJM}}(\bar{1}2\bar{3}4\bar{5}6\bar{7}8) + A^{\mathrm{ABJM}}(\bar{1}2\bar{3}6\bar{5}4\bar{7}8) + A^{\mathrm{ABJM}}(\bar{1}2\bar{5}4\bar{3}6\bar{7}8) \\
&\quad + A^{\mathrm{ABJM}}(\bar{1}2\bar{5}6\bar{3}4\bar{7}8) - A^{\mathrm{ABJM}}(\bar{1}8\bar{3}4\bar{7}6\bar{5}2) - A^{\mathrm{ABJM}}(\bar{1}8\bar{3}6\bar{7}4\bar{5}2) \\
A^{\mathrm{BLG}}(2\bar{3}\bar{5}\bar{7}8;\bar{1}46) &= -A^{\mathrm{ABJM}}(\bar{1}2\bar{3}4\bar{5}6\bar{7}8) - A^{\mathrm{ABJM}}(\bar{1}2\bar{3}6\bar{5}4\bar{7}8),
\end{aligned}$$

Maximal susy is extremely special in D=3

- (BLG) the only three-algebra theory that has amplitude relations.
- ($\mathcal{N} = 16$ sugra) the only supergravity that allows a double-double copy

Conclusion

- The scattering amplitude of ABJM is given by integrals over cells in the positive orthogonal grassmannian $\mathrm{OG}_{\mathrm{k}+}$
- Each cell in the positive orthogonal grassmannian $\mathrm{OG}_{\mathrm{k}+} \rightarrow$ cell $\mathrm{Gr}(\mathrm{k}, 2\mathrm{k})_{+}$.
- The canonical form has logarithmic singularity at $\partial\mathrm{OG}_{\mathrm{k}+}$ (Not in dlog form)
- The combinatorics have the same features with positive grassmannian.
- Mysterious BCJ relations for BLG partial amplitudes.
- Color-Kinematics in three-dimensions pin-points $\mathcal{N} = 16$ SUGRA as special

Conclusion

- Can we prove that the IR-divergences are the same ?
- Can we find a polytope picture (extendable to $\mathcal{N} < 6$)
- Twistor string theory ? see Oluf Engelund, Radu Roiban
- String theory derivation for BLG amplitudes $\rightarrow$ amplitude relations ?
- Is $\mathcal{N} = 16$ finite?