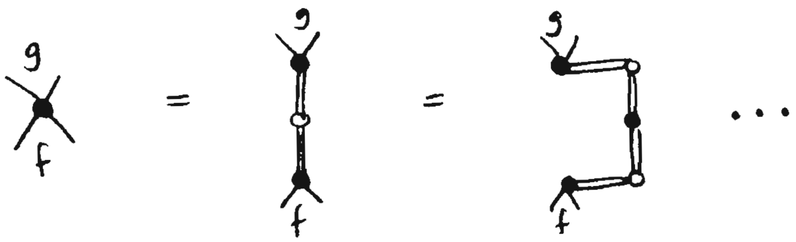
Conformal Field Theories and Twistor Diagrams

In an earlier article (**TN 23**) I emphasised the vital importance of locating a prescriptive theory of fundamental physics of which twistor diagrams could be the evaluative calculus (in analogy to Feynman diagrams). I commented on the appearance of the vertices

in twistor diagrams for massless electroweak theory, hazarding the suggestion that such diagrams might be generated systematically by a combinatorial rule based on such vertices. Such a rule, if it existed, should then be derived from a deeper theory in analogy to the derivation of the Feynman rules from an interaction Lagrangian.

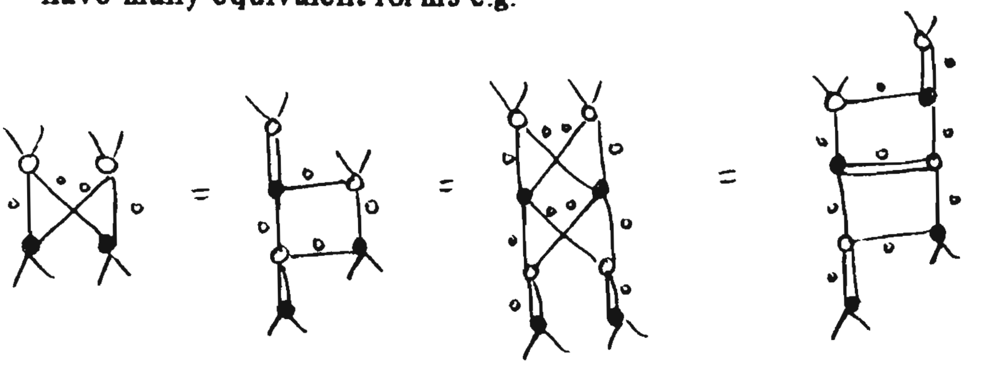
Despite the suggestive features of these twistor diagrams, however, it was not possible actually to establish any such combinatorial rule. There is, furthermore, a prominent feature of twistor diagrams distinguishing them from Feynman diagrams, namely that for any particular amplitude there are many twistor diagram representations. This suggests that the analogy with Feynman diagrams may be indirect.

As examples: even for the zero-order interaction we have



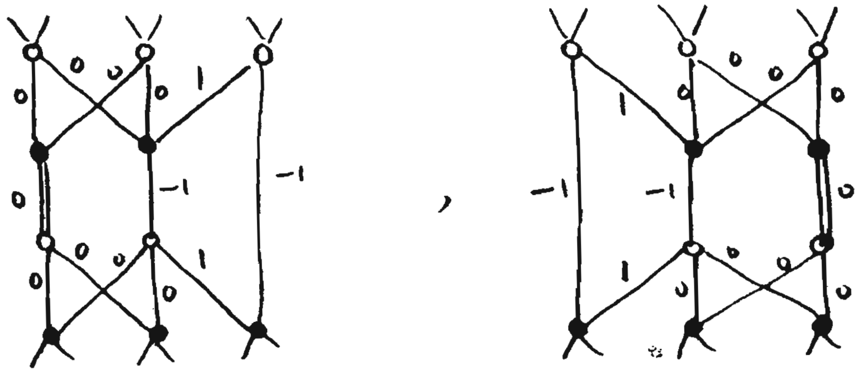
(A)

which is enough in itself to suggest that the "order" of a diagram cannot be defined in terms of the number of its vertices. At the first order level we have many equivalent forms e.g.



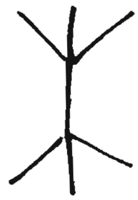
etc. etc.

Likewise, if we consider the higher-order diagrams described in **TN 25**; we note the equivalence of



etc. (B)

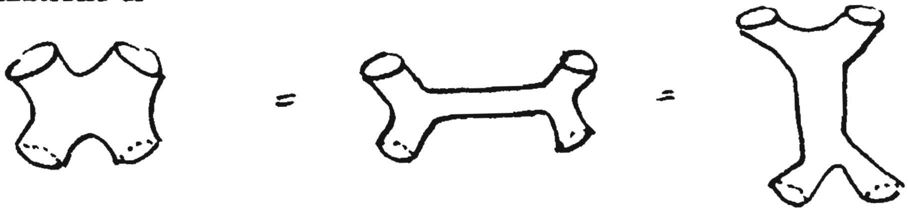
All of these correspond to the Feynman diagram for second-order ϕ^4 scattering



but none of them exhibit the actual symmetry of the amplitude. Reference to that article will show many other examples.

Now R.P. did in fact suggest long ago that there was some similarity to be seen between twistor diagrams and the planar diagrams of the Veneziano dual model, originally devised in the context of describing the strong interaction. As is well known, the identity

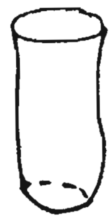
of the planar diagrams can be interpreted in terms of string interactions: both are realizations of



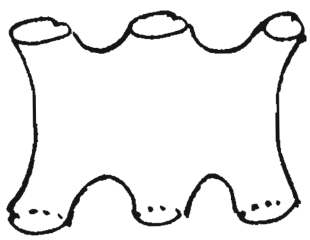
We can therefore ask the question: is there some analogous structure in *twistor* geometry such that the many different equivalent twistor diagrams can be interpreted as different ways of evaluating an amplitude properly defined on that structure? This question could have been asked at any time in the last 15 years or so, and it is hard to see why we have not addressed it before. However, our recent exposure to conformal field theories, with its emphasis on *complex manifold* structure, has not only prompted the question more acutely than before but has stimulated a specific suggestion for what this structure could be (see Mike Singer, Florence Tsou, Roger Penrose, this **TN**): namely (i) the interpolation of complex manifolds between copies of **PN** and (ii) in some way specifying free in- and out-fields on those copies of **PN**, (iii) in some way analytically continuing such data across the interpolating manifolds and then combining them to give a natural functional of the in- and out-states.

Let us adopt M.A.S.'s pictures for this structure. We shall adopt the interpretation in which the boundaries of the picture are associated with *one-particle* states. In the first instance these are *massless* fields, so that an appropriate H^1 in one twistor variable is prescribed on each **PN** boundary piece. [However, there is room in this scheme, following R.P.'s suggestion, for a two-twistor or n-twistor object to be prescribed on a boundary. This idea opens up a new view of how the twistor representation of a massive one-particle state by n twistors can differ essentially from a massless n-particle state - a question hitherto puzzling fro the point of view of twistor diagram theory.]

We are thus led to hazard the suggestion that all the inner product diagrams (A) might be seen as different evaluations of something like



and the diagrams (B) as evaluations of something of form



(an object in which the true symmetry would be manifest, even though that symmetry is broken when choosing a specific evaluation *via* a twistor diagram.)

If this were so then we would replace the idea of a sum over graphs defined by vertices by a sum over all interpolating complex manifolds. *This* would become the analogy to the summing over Feynman diagrams, and we should then go on to seek some fundamental theory explaining *this* generating rule.

As yet we have no theory that yields a correspondence between the Singer pictures and twistor diagrams. But there are general reasons why one might be hopeful:

(1) Note that (at least in the first instance) we are looking for a twistor-based theory which gives a new description of an essentially well-known flat-space theory of massless fields. We are translating interactions which are described in space-time as interactions at a point. But points are ***extended*** objects in **T**. So we should always have ***expected*** something "stringlike" in **T** to emerge.

(2) Again, note that (at least in the first instance, and modulo divergence problems), we know the functionals of free fields that we are looking for - holomorphic conformal invariant linear functionals with various symmetries. If we can find ***any*** way of deriving functionals with these features from a theory based on Singer pictures, then there seems an excellent chance that they will be the right ones.

(3) In looking for a correspondence between Singer pictures and twistor diagrams, we might look first at the very simplest case - the inner product diagrams (A). For a further simplification we might further look at the analogous spinor integrals. Of these, the very simplest example is

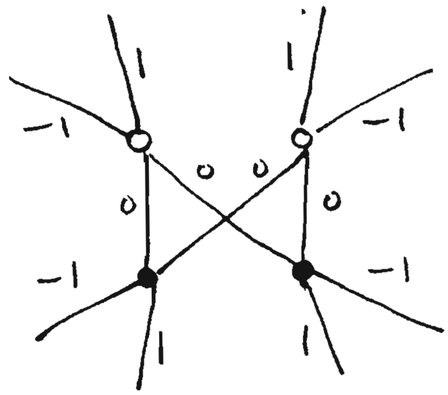
$$\oint f(\lambda)g(\lambda)\,D\lambda = \oint f(\lambda)(\lambda.\mu)^{-1}g(\mu)\,D\lambda\mu = \oint f(\lambda)(\lambda.\mu)^{-1}(\mu.\nu)^{-1}g(\nu)\,D\lambda\mu\nu$$

etc. etc.

These are large-dimensional contour integrals in various products of P^1s But they could be ***re-interpreted*** as specifying the glueing together of various pieces of P^1s by making the identifications $\lambda = \mu$, $\mu = \nu$, etc., so that each integrals is really being done on the same P^1 manifold, described in different ways.

Although this is a hopeful line of thought, I must say that at present I have no idea how it can be generalised to other homogeneities, or to twistor space in a way that naturally brings in the dual spaces.

Lastly, I refer to my third article in **TN 25**. There it was argued that the twistor diagrams that traditionally have been considered, and such as have been written down above, are not the fundamental objects. They should be thought of as ***periods*** of the more fundamental but as yet not very well defined integrals given by (e.g.)



These are the objects which are glued together to make twistor diagrams for higher-order amplitudes, i.e. correspond to the combination of the *off-shell* Feynman propagators in Feynman diagrams. One takes various possible periods of these integrals to obtain the amplitudes that arise when the external legs are specified to correspond to *free* in- or out-fields in the various possible channels. Thus I suggest that *these* are the objects that should correspond to the pieces of manifold that are in some sense glued together to build up higher-order Singer pictures. It seems to me therefore that a Singer picture should turn out to specify not an amplitude, but some functional (perhaps not very well defined) whose various *periods* would give the amplitudes in the various different possible channels. Note that inhomogeneity (the "k") and logarithmic propagators were essential in defining these "off-shell" diagrams. I suggest that corresponding [non-obvious] structures would have to be appear in any theory of manifolds which makes sense of the Singer pictures.

Thanks to Mike Singer, Roger Penrose and Florence Tsou -

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