## A Symplectic Penrose Transform?

The Penrose transform on complex homogeneous spaces is well worked out, now. One might ease the requirement of homogeneity by considering symplectic, Käehler manifolds $Z$ with a symplectic $G_0$-action; $G_0$ is a compact Lie group. These yield the "twistor space" side of a double fibration as follows: let $G$ be the complexification of $G_0$ and $X = G/P$, $P \subset G$ parabolic. So $X$ is homogeneous and there is a moment map $\mu_X : X \longrightarrow \mathfrak{g}_0^*$, given a line bundle $\mathcal{L}$ on $X$. For $x \in X$ let $\hat{x} = i V_x$ where $V_x$ is the vector in $\mathfrak{g}_0$ corresponding to $\mu_X(x)$ under the Killing form. Let $\mu_Z : Z \longrightarrow \mathfrak{g}_0^*$ be a moment map, also, and let

$$f(z) = \langle \hat{x} \cdot \mu_Z(z) , \mu_Z(z) \rangle \qquad (\langle \, , \rangle : \text{Killing form})$$

It turns out that the maximum of $f$ (which is real, by virtue of $\hat{x}$ being hermitean) is achieved on an even dimensional subvariety of $Z$ (which might be called the coherent subvariety* of $Z$ corresponding to $x$). It appears this subvariety is a complex subvariety of $Z$. Indeed, if $Z = G/Q$ (eg., $Y = \mathbb{PT}$, $X = \mathbb{M}$) then this subvariety $L_x$ is exactly the corresponding variety. So we have a "double fibration"

$$\begin{tikzcd} & Y \arrow[dl, "\eta"'] \arrow[dr, "\tau"] & \\ Z & & X \end{tikzcd}$$

$Y \longleftarrow$ the "correspondence space".

If the symplectic structure on $Z$ is integral $Z$ has a natural complex line bundle $\mathcal{L}_Z$ (it would be $\mathcal{O}(1)$ for $Z = \mathbb{CP}_3$) and one ought to be able to compute the Penrose transform for $\mathcal{L}_Z^{\otimes p}$

(* this terminology because the construction follows that of coherent states in geometric quantization)

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## 2 dimensional conformal invariants

Let $\Sigma$ be any complex curve with distinguished volume form, thought of as a real manifold. The methods of Ochiai & many others (see my thesis) show that $\exists$ a $\hat{B}$-principal bundle $\hat{P} \longrightarrow \Sigma$ where $B$ is the subgroup of upper triangular matrices in $PSL(2,\mathbb{C})$; If $\Sigma$ is spin, this extends to $P \longrightarrow \Sigma$, $B$-principal, $B$ = upper $\Delta$ in $SL(2,\mathbb{C})$. The Cartan connections on this structure are not unique (as in higher dimensions than 3) but parameterized by sections $\Phi$ of the quadratic forms $\Omega^{\otimes 2}$ of $\Sigma$; there ought to be a way of using Verma modules for the Virasoro algebra to construct invariants here. If for some reason $\Phi$ is given then the Verma module theory of $\mathfrak{sl}(2,\mathbb{C})$ will yield differential invariants. If $\Sigma$ is the lift of a null geodesic in curved $M$ to projectivized spin & spin' bundle $\mathbb{F}_{123}$ then $\Phi$ is determined to be $\Phi^{ABA'B'}\pi_{A'}\pi_{B'}\mu_A\mu_B$ and invariants of the conformal structure of $M$ result — see [1], [2]

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[1] Rod Gover: thesis expected soon.
[2] MGE: TN 20, p40.