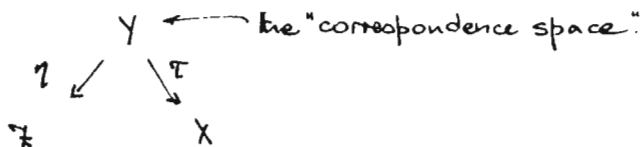


A Symplectic Penrose Transform?

The Penrose transform on complex homogeneous spaces is well worked out, now. One might ease the requirement of homogeneity by considering symplectic, Kähler manifolds Z with a symplectic G_0 -action; G_0 is a compact Lie group. These yield the "twistor space" side of a double fibration as follows: let G be the complexification of G_0 and $X = G/P$, $P \subset G$ parabolic. So X is homogeneous and there is a moment map $\mu_X: X \rightarrow \mathfrak{g}_0^*$, given a line bundle \mathcal{L} on X . For $x \in X$ let $\hat{x} = iV_x$ where V_x is the vector in \mathfrak{g}_0 corresponding to $\mu_X(x)$ under the Killing form. Let $\mu_Z: Z \rightarrow \mathfrak{g}_0^*$ be a moment map, also, and let

$$f(z) = \langle \hat{x} \cdot \mu_Z(z), \mu_Z(z) \rangle \quad (\langle \cdot, \cdot \rangle : \text{Killing form})$$

It turns out that the maximum of f (which is real, by virtue of \hat{x} being hermitean) is achieved on an even dimensional subvariety of Z (which might be called the coherent subvariety* of Z corresponding to x). It appears this subvariety is a complex subvariety of Z . Indeed, if $Z = G/Q$ (eg. $Y = \mathbb{P}^1$, $X = \mathbb{M}$) then this subvariety h_x is exactly the corresponding variety. So we have a "double fibration"



If the symplectic structure on Z is integral Z has a natural complex line bundle \mathcal{L}_Z (it would be $\mathcal{O}(1)$ for $Z = \mathbb{C}P^1$) and one ought to be able to compute the Penrose transform for $\mathcal{L}_Z^{\otimes p}$

(* this terminology because the construction follows that of coherent states in geometric quantization)

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2 dimensional conformal invariants

Let Σ be any complex curve with distinguished volume form, thought of as a real manifold. The methods of Ochiai & many others (see my thesis) show that \exists a \mathbb{B} -principal bundle $\hat{P} \rightarrow \Sigma$ where \mathbb{B} is the subgroup of upper triangular matrices in $\mathbb{P}SL(2, \mathbb{C})$; If Σ is spin, this extends to $P \rightarrow \Sigma$, \mathbb{B} -principal, $\mathbb{B} =$ upper Δ in $SL(2, \mathbb{C})$. The Cartan connections on this structure are not unique (as in higher dimensions than 3) but parameterized by sections Φ of the quadratic forms $\Omega^{\otimes 2}$ of Σ ; there ought to be a way of using Verma modules for the Virasoro algebra to construct invariants here. If for some reason Φ is given then the Verma module theory of $\mathfrak{sl}(2, \mathbb{C})$ will yield differential invariants. If Σ is the lift of a null geodesic in curved M to projectivized spin & spin' bundle $F_{1,2}$ then Φ is determined to be $\Phi_{ABAB} \tau_A \tau_B \mu_A \mu_B$ and invariants of the conformal structure of M result — see [1], [2]

[1] Rod Gover: thesis expected soon.
[2] NGE: \mathbb{R}^4 20, p 40.

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