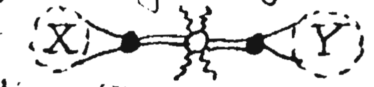
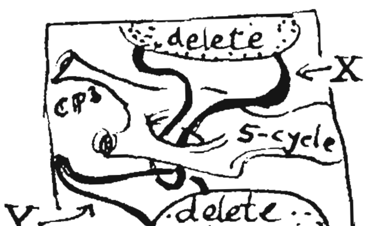
Holomorphic Linking: Postscript

If we formally integrate out the  in the double twistor transform  to get , and represent each 1-function ,  locally by a pair of equations $f=0=g$, $h=0=j$, respectively, then we get $\mathbf{L} \doteq \sum \oint \frac{f'(z)\, g'(z)\, h'(z)\, j'(z)\, d^4 z}{f(z)\, g(z)\, h(z)\, j(z)}$ ($f' = \frac{\partial f}{\partial z}$ etc.)

i.e. $\mathbf{L} \doteq \sum \oint \frac{df \wedge dg \wedge dh \wedge dj}{f g h j} = \sum (\pm (2\pi i)^4)$, where the "$\Sigma$" tots up all the needed pieces for the two 1-functions. In the case where X and Y are algebraic curves, we can use a dot product of the two H^1's to get an H^3 elem. & evaluate this 3-function over $\mathbb{PT}$ to get a <u>canonical</u> value for $\mathbf{L}$ (i.e. indep. of a selection of contours). If we normalize $\mathbf{L}$ so that $\mathbf{L}=1$ for two lines in $\mathbb{CP}^3$, then we get $\boxed{\mathbf{L} = xy}$ where x and y are the orders of $\mathbf{X}$ and $\mathbf{Y}$, respectively (number of points they meet a generic plane). This is one <u>quarter</u> of the ("standard") linking number which would be **1** for two linking circles. (This result can be obtained by specializing $\mathbf{X}$ to x lines and $\mathbf{Y}$ to y lines.) A possible suggestion for the more general holomorphic linkings (e.g. distinguishing  from  !) might be to find a real 5-cycle separating $\mathbf{X}$ from $\mathbf{Y}$ (in $\mathbb{CP}^3$) and such that $\exists$ 1-functions defining $\mathbf{X}$ and $\mathbf{Y}$ whose domains intersect in a region containing the 5-cycle.



Each 1-function is a 1-form projectively and a 2-form non-projectively. Use the <u>non</u>-projective form, and <u>cup</u> product to obtain a (2+2)-form (1+1)-function, i.e. element of $H^2(d\Omega^3)$ (since closed). We can evaluate this on the non-projective 6-cycle which is the circle bundle over the 5-cycle in the S^7 of "unit circles" over $\mathbb{CP}^3$ (in $\mathbb{C}^4$). I don't know if this gives significantly more than we had before.

<u>REMARK</u> I find the relation between holomorphic linking and twistor diagrams intriguing. Linking (at least, <u>real</u> linking) is a <u>combinatorial</u> thing. Could there be a deep combinatorial aspect to twistor diagram theory? Could this relate to the original motives lying behind spin-network theory?

~ Roger Penrose