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A conformally invariant connection

Toby Bailey

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This note is a postscript to the last section of my article in TN26 on the conformally invariant connection associated to a direct sum decomposition of one of the spin bundles. The general result lying behind the observations in that article , stated for convenince in the holomorphic category, is:

Theorem: *Let $\mathcal{M}$ be a complex conformal manifold with conformal metric g_{ab} and a given tensor field $J_a{}^b$ with $J_{(ab)} = 0$ and $J_a{}^c J_c{}^b = -\delta_a{}^b$. Then there exists a unique torsion-free connection ∇_a satisfying*

$$\nabla_a J_b{}^a = 0$$

$$\nabla_a g_{bc} = X_a g_{bc} \quad \text{for some } X_a.$$

The second condition is simply that the conformal metric is preserved.

If one is given a direct sum decomposition of $\mathcal{O}^A$ in a complex space-time then $J_a{}^b = i(o_A \iota^B + \iota_A o^B)\epsilon_{A'}{}^{B'}$, where o_A, ι_A constitute a spin-frame defining the decomposition, satisfies the above conditions and the resulting connection is given in components by RPs 'conformally invriant edth and thorn' operators.

The significance of the rather strange condition on the derivitive of $J_a{}^b$ which defines the connection is unclear, and work continues on the use of this connection in type D conformal space-times and related areas.

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